STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- Why Engineer needs economics concepts?
- The terminologies used in the engineering economic analysis
- Principle of Engineering Economics.
- To develop the cash flow diagram
- How economic system is carried out?

1.1 Definition of Economics

Definition 1

Economics is the social science that examines how people choose to use limited or scarce resources in attempting to satisfy their unlimited wants. (N. Gregory Mankiw)

Definition 2

Economics is a science which studies human behavior as a relationship between ends and scarce means which have alternative uses. (Lionell Robbins)

Why Engineering Economics?

The field of engineering economy is concerned with the systematic evaluation of the benefits and costs of the projects involving engineering design and analysis. Engineering economy quantifies the benefits and costs associated with engineering projects to determines if they make (or save) enough money to warrant their capital investment. In manufacturing or construction, engineering is involved in every detail of a product's production (about 85%) from conceptual design to distribution. Engineers must decide if the benefits of a project exceed its costs and must make this comparison in a unified framework. The framework within which to make this comparison is the field of engineering economics.

In the development of any product, a company's engineers are called upon to translate an idea into reality. A firm's growth and development largely depends upon a constant flow of ideas for new products, and for the firm to remain competitive, it has to make existing products better or produce them at a lower cost. Traditionally, a marketing department would propose a product and pass the recommendation to the engineering department. The engineering department would work up a design and pass it on a manufacturing, which would make the product. With this type of product development cycle, a new product normally takes several months (or even years) to reach a market. Decisions made by the engineers are commonly the result of choosing one alternative over another. Decisions often reflect a person's educated choice of how to best invest funds, also called capital. The amount of capital is usually restricted, just as the cash available to an individual is usually limited. The decision of how to invest the capital will invariably change the future, hopefully for the better: that is, it will be value adding. Engineers play a major role in capital investment decisions based on their analysis, synthesis, and design efforts. The factors considered in making the decision are a combination of economic and noneconomic factors.

Hence we can define Engineering Economics in a following different way:

Definition 1"Engineering economics is the application of economic techniques to the evaluation of design and engineering alternatives. The role of engineering economics is to assess the appropriateness of a given project, estimate its value, and justify it from an engineering standpoint". (Dr. John M.Watts)

Definition 2"Engineering economics deals with the methods that enable one to take economics decision towards minimizing the cost or maximizing benefits to business organization".

Definition 3"Engineering economics deals with the concepts and techniques of analysis useful in evaluating the worth of systems, products, and services in relation to their costs".

Definition 4"Engineering economy involves formulating, estimating, and evaluating the economic outcomes when alternatives to accomplish a defined purpose are available. Another way to define engineering economy is as a collection of mathematical techniques that simplify economic comparison".

Knowing how to correctly apply these techniques is especially important to engineers, since virtually any project will affect costs and/or revenues.

1.2 Origin of Engineering Economy

Cost considerations and comparisons are fundamental aspects of engineering practice. However, the development of engineering economy methodology, which is now used in all engineering work, is relatively recent. This does not mean that, historically, costs were usually overlooked in engineering decisions. However, the perspective that ultimate economy is a primary concern to the engineer and the availability of sound techniques to address this concern differentiates this aspect of modern engineering practices from that of the past.

A pioneer in the field was Arthur M. Wellington, a civil engineer, who in the latter part of the nineteenth century specifically addressed the role of economic analysis in engineering projects. His interest was railroad building in USA. His work was followed by Eugene Grant who published the first edition of his textbook which was the milestone in the development of engineering economy as we know it today. He placed emphasis on developing an economic point of view in engineering. In 1942 Woods and DeGarmo wrote the first edition of this book, later entitled Engineering Economy.

1.3 Role of Engineering Economy

People make decisions: computers, mathematics and other tools do not make. The techniques and models of engineering economy assist people in making decisions. Since decision affect what will be done, the time frame of engineering economy is primarily the future. Therefore, numbers used in an engineering economic analysis are best estimates of what is expected to occur. These estimates often involve the three essential elements: cash flows, time of occurrence and interest rate. These estimates are about future, and will be somewhat different than what actually occurs, primarily because of changing circumstances and unplanned for events. In other words stochastic nature of estimate will likely make the observed value in the future differ from the estimate made now. Especially, sensitivity analysis is performed during the engineering economic study to determine how the decision might change based on varying estimate (discussed in chapter 8).

Engineering economy can be used equally to analyze outcomes have met or not met a specified criterion, such as rate of return requirement (discussed in chapter 4). There is an important procedure used to address the development and selection of alternatives. Commonly referred as the problem-solving approach or the decision making process, the steps in the approach follow:

1. Understand the problem and define the objective.
2. Collect relevant information.
3. Define the feasible alternative solutions and make realistic estimates.
4. Identify the criteria for decision making using one or more attributes.
5. Evaluate each alternative, using sensitivity analysis to enhance the evaluation.
6. Select the best alternative.
7. Implement the solution and monitor the results.

Strategic Economic Decisions

Once project ideas are identified, they are typically classified as:

1. Equipment and process selection
2. Equipment Replacement
3. New product development and product expansion
4. Cost reduction, and
5. Service improvement

1.4 Principles of Engineering Economics

The principle of engineering economics can be highlighted in the seven points as below:

Principle 1

Develop the Alternatives: The choice is among alternatives. The alternatives need to be identified and then defined for subsequent analysis.

Principle 2

Focus on the Differences: Only the differences in expected future outcomes among the alternatives are relevant to their comparison and should be considered in the decision.

Principle 3

Use a Consistent Viewpoint: The prospective outcomes of the alternatives, economic and other, should be consistently developed from a defined viewpoint (perspective).

Principle 4

Use a Common Unit of Measure: Using a common unit of measurement to enumerate as many of the prospective outcomes as possible will make easier the analysis and comparison of the alternatives.

Principle 5

Consider All Relevant Criteria: Selection of preferred alternative requires the use of criteria or several criteria. The decision process should consider both the outcomes enumerated in the monetary unit and those expressed in some other unit of measurement or made explicit in the descriptive manner.

Principle 6

Make Uncertainty Explicit: Uncertainty is inherent in projecting the future outcomes of the alternatives and should be recognized in their analysis and comparison.

Principle 7

Revisit the Decision: Improved decision making results from an adaptive process to the extent practicable, the initial projected outcomes of the selected alternative should be subsequently compared with actual results achieved.

1.5 Essential Economics Terminology

1. **Annuity:**
- An amount of money payable to a beneficiary at regular intervals for a prescribed period of time out of a fund reserved for that purpose.
- A series of equal payments occurring at equal periods of time
  - Amount paid annually/monthly/weekly etc., including reimbursement of borrowed capital and payment of interest.

2. **Assets:** An economic resource of entity (including money resources, physical resources, and intangible resources).

3. **Breakeven point:**
- A graphic representation of the relation between total income and total costs for various levels of production and sales indicating areas of profit and loss.
- A point where the organization is in no gain and no loss state.

4. **Capital:**
   - The financial resources involved in establishing and sustaining an enterprise or project.
   - A term describing wealth which may be utilized to economic advantage. The form that this wealth takes may be as cash, land, equipment, patents, raw materials, finished products, etc.
5. **Cash flow:** The statement showing actual amount coming into the firm and/or going out of the firm.
6. **Capital recovery:** It is the annual equivalent cost of capital cost.
7. **Discount rate:** The interest rate used to calculate the present value of the future cash flows.
8. **Decision making:** A program of action undertaken as a result of established policy to influence the final decision.
9. **Decision making under certainty:** Simple decisions that assume complete information and no uncertainty connected with the analysis of the decisions.
10. **Decision making under uncertainty:** Decision for which the analyst elects to consider several possible futures, the probabilities of which cannot be estimated.
11. **Decisions under risk:** A decision problem in which the analyst elects to consider several possible futures, the probabilities of which can be estimated.
12. **Depreciation:**
   - Decline in value of a capitalized asset.
   - A form of capital recovery applicable to a property with two or more years' life span, in which an appropriate portion of the asset's value is periodically charged to current operations.
13. **Economic life:** The timeframe an asset will be economically useful.
14. **Economic efficiency:** It is the ratio of output to input of a business system.

Economic efficiency (%) = Output/Input*100

= Worth/ Cost *100

15. **Interest:** Interest is the fee that is charged for use of someone else's money. The size of the fee will depend upon the total amount of money borrowed and the length of time over which it is borrowed.

<u>Simple interest</u>: It is defined as the fixed percentage of the principal (the amount of money borrowed) multiplied by the life of the loan.

<u>Compound interest</u>: The type of interest that is periodically added to the amount investment (or loan) so that subsequent interest is based on the cumulative amount.

16. **Inflation:** An increase in the average price paid for goods and services bringing about reduction in the purchasing power. The inverse of inflation is *deflation.*

17. **Intangibles:**
- In economic studies, conditions or economy factors that cannot be readily evaluated in quantitative terms as in money.
- In accounting, the assets that cannot be reliably evaluated (e.g., goodwill, social values).

18. **Labor:** The capacity of human effort (both mind and muscles) available for use in producing goods and services.

19. **Opportunity cost:**
- The value of benefits sacrificed in selecting a course of action among alternatives.
- The value of the next best opportunity foregone by deciding to do one thing rather than another.

20. **Salvage Value:** Receipt at project termination for sale or transfer of the equipment (can be a salvage cost).

21. **Time value of money:** Since money has the ability to earn interest, its value increases with time. Hence it is the relationship between interest and time.

22. **Utility:** Satisfaction that a consumer obtains from goods and services that are consumed. It is a measure of satisfaction.

1.6 Definition of cash flow

Cash flow is the stream of monetary (Rupees) values—costs (inputs) and benefits (outputs)—resulting from a project investment. The analysis of events and transactions that affects the cash position of company is termed as cash flow. A cash flow is the difference between total receipts (inflows) and total cash disbursement (outflows) for a given period of time. It is the statement that shows the actual amount coming into firm or going out of the firm.

Cash Inflows: Actual rupees coming into a firm.

Cash outflows: Actual rupees going out from the firm.

Cash Flow diagrams (CFD)

The costs and benefits of engineering projects over time are summarized on a cash flow diagram (CFD). Specifically, CFD illustrates the size, sign, and timing of individual cash flows, and forms the basis for engineering economic analysis. It is difficult to solve a problem if you cannot see it. The easiest way to approach problems in economic analysis is to draw a picture. The picture should show three things:

1. A time interval divided into an appropriate number of equal periods

2. All cash outflows (deposits, expenditures, etc.) in each period.

3. All cash inflows (withdrawals, income, etc.) for each period.

Unless otherwise indicated, all such cash flows are considered to occur at the end of their respective periods.

Drawing a Cash Flow Diagram

- In a cash flow diagram (CFD) the end of period t is the same as the beginning of period (t+1).

- Beginning of period cash flows are: rent, lease, and insurance payments
- End-of-period cash flows are: O&M, salvages, revenues, overhauls
- The choice of time zero is arbitrary. It can be when a project is analyzed, when funding is approved, or when construction begins
- One person's cash outflow (represented as a negative value) is another person's inflow (represented as a positive value)
- It is better to show two or more cash flows occurring in the same year individually so that there is a clear connection from the problem statement to each cash flow in the diagram
- Arrow lengths are approximately proportional to the magnitude of cash flow.

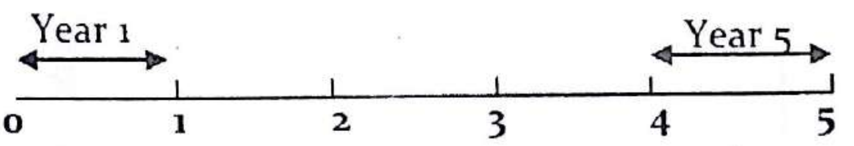
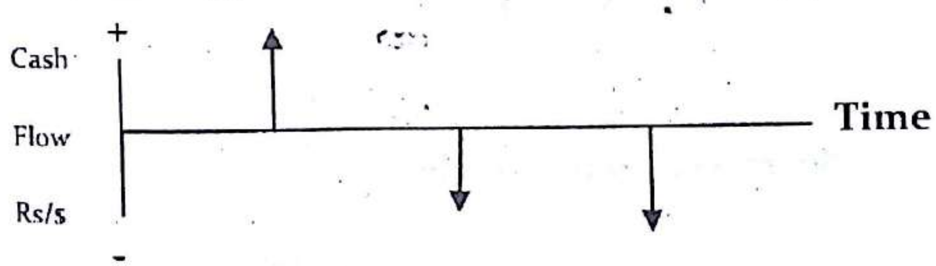
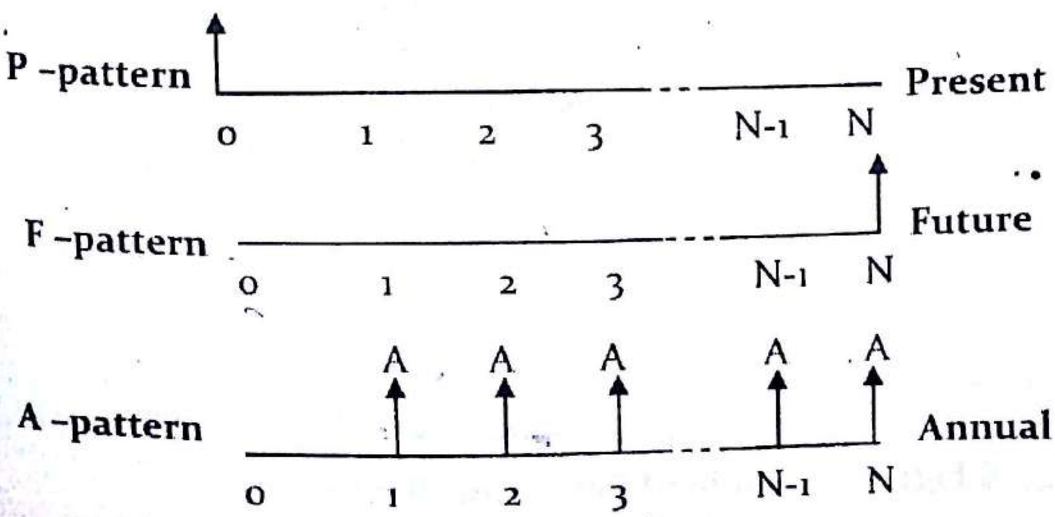


Fig1.1: A typical cash flow time scale for 5 years





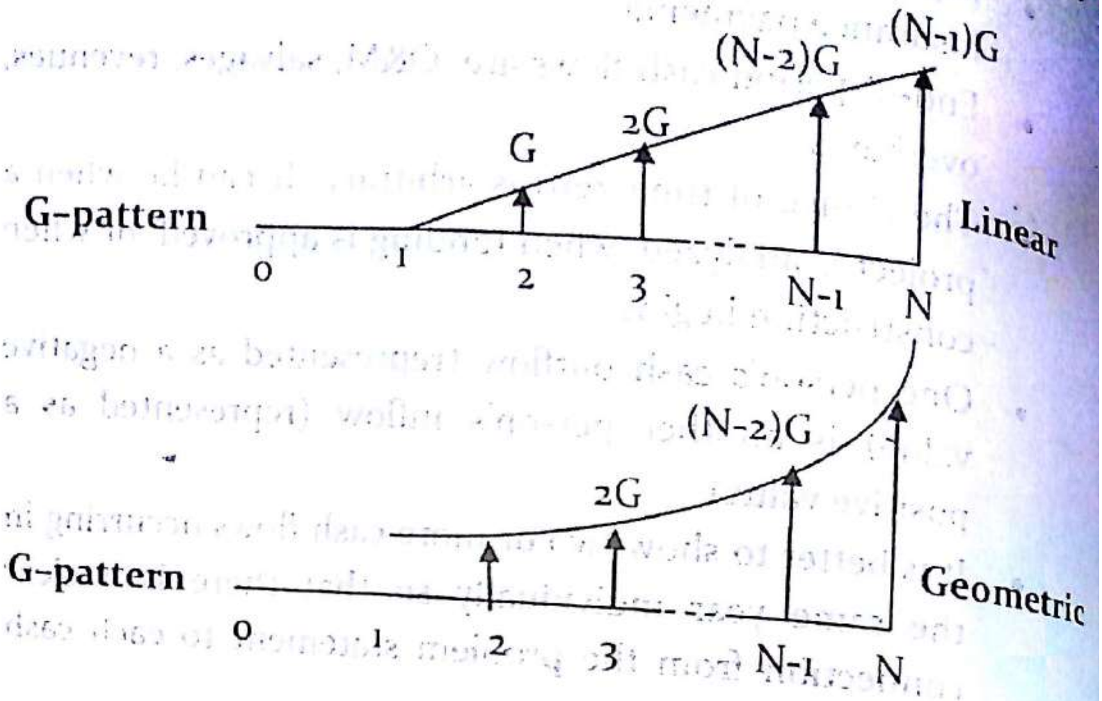


Fig 1.3: Pattern of Cash Flow

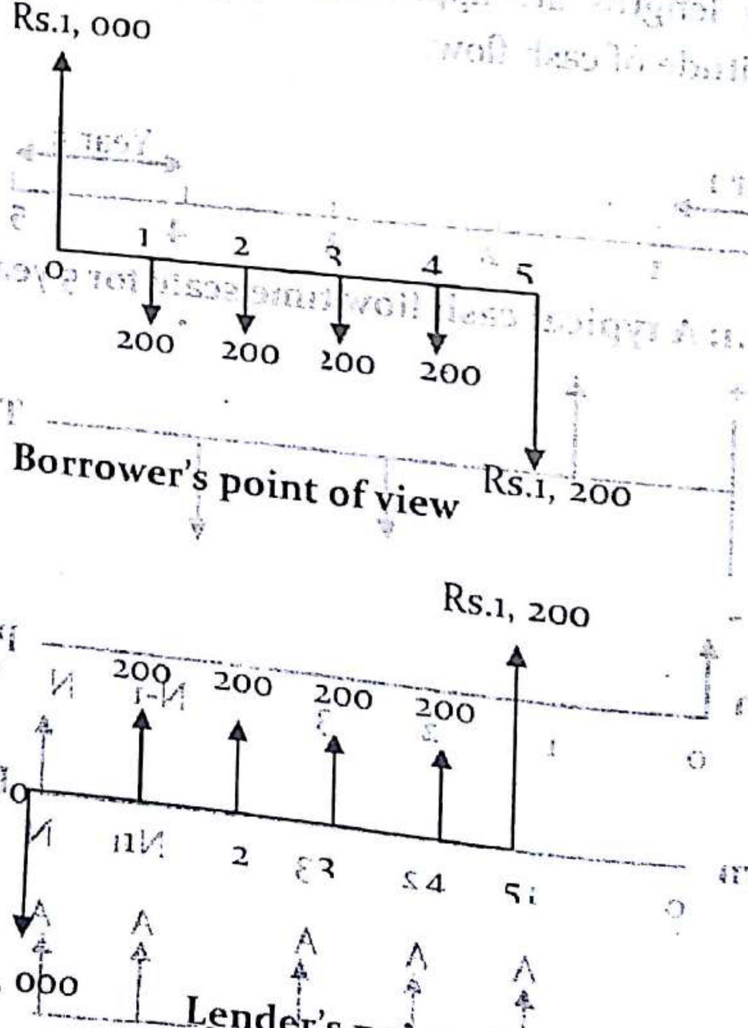


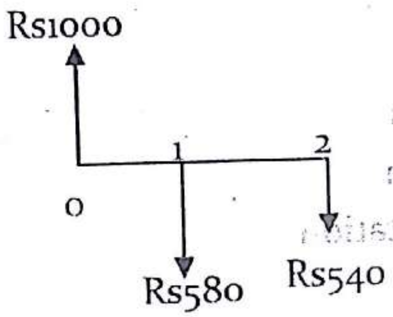
Fig1.4: Example of Cash flow diagrams

Example 1.1

A man borrowed Rs. 1,000 from a bank at 8% interest. Two end-of-year payments: at the end of the first year, he will repay half of the Rs.1000 principal plus the interest that is due. At the end of the second year, he will repay the remaining half plus the interest for the second year.

Cash flow for this problem is:

End of year	Cash flow
0	+Rs 1, 000
1	-Rs 5, 80 (-Rs500 - Rs80)
2	-Rs 5, 40 (-Rs500 - Rs40)



1.7 Economic System

Economic system is the institutional framework within which a society or country carries on its economic activities. Mainly, there are three types of systems:

1. **Private enterprise system (Capitalistic Economic System)**
2. **Pure socialistic system**
3. **Combination of both (Mixed Economic System)**

1. Private enterprise system (Capitalistic Economic System)

Under this system, all firms, factories and other means of productions are the property of private individuals and organizations. They are free to use them with a view to making profit. What to produce how to produce and for whom to produce, all these central problems of economic are settled by the free working of the forces of demand and supply.

Features

- Right of private property
- Freedom of enterprise
- Freedom of choice by consumers
- Profit motive
- Class conflict

Merits

- Individual initiative
- Perfect competition
- Dynamic economy

Demerits

- Inequality of incomes
- Inefficient production
- Monopoly and exploitation
- Unemployment

2. Pure socialistic system

In pure socialistic system there is no private property. Resources, goods and services are owned and controlled by the government. Production takes place in government enterprises and the government specifies the conditions under which exchange can occur.

Features

- Social ownership of means of production
- No private enterprise
- Economic equality
- Equality of opportunity
- Economic planning
- Social welfare and social security.

Merits

- In the absence of private profit, production will be shifted from more profitable goods to more useful goods.
- Many things, consumption of which is considered essential for health and efficiency, may be supplied free or below cost.
- Socialist economy will prevent cyclical fluctuations in business activity and will bring about smooth working of the economy.

Demerits

- Bureaucratic running of the system
- It will lead to concentration of both political and economic power in the hands of government.
- There is no proper basis of cost calculation and in the absence of such a basis, the economy cannot function in an efficient manner or allocate the resources in the best possible way.

3. Combination of both (Mixed Economic System)

The private enterprise system is decentralized where as socialistic system is highly centralized. In present days, economy is the mixture of socialism and private enterprises. According to mixed economy, some part of an economy's output will be produced by the profit oriented private sectors and another part will be produced in a socialistic manner by the public sector. There are also nonprofit sectors like hospitals, schools etc.

Features

- Co-existence of the public and private sectors
- Role of government directions
- Government protection of labor
- Control of monopoly

STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- To understand the concept of simple interest and compound interest
- To understand the concept of economic equivalence
- To derive the formula for compound interest
- To derive the formula for the discrete cash flow and discrete compounding
- To derive the formula for continuous compounding and continuous cash flow
- To derive the formula for gradient series (linear and geometric)
- To solve the numerical problems of the discrete and continuous compounding and gradient series

3.1 Time value of Money

Time value of money is defined as the time-dependent value of money stemming both from changes in purchasing power of money (inflation or deflation) and from the real earning potential of alternative investments over time. Since money has the ability to earn interest, its value increases with time. Hence it is the relationship between interest and time.

Interest

Most of us are familiar in a general way with the concept of interest. We know the money left in the savings account earns interest so that the balance over time is greater than the sum of the deposits. Whenever we go for any investment, we will have to consider the following three factors:

(a) Liquidity

Once it is invested, it is not so easy to convert it to cash and when needed immediately, we will not be able to spend on another project or on other financial expenses. In other words, it is the reward for not being able to use your money while you are holding the stock or mortgage or promise.

(b) Risk premium

There is always a certain degree of risk associated with any financial investment. For example, if you lend someone Rs 1,000, it is not sure that you may get it back either because of his nature or market scenario. The situation is worse in case when you are making investment on businesses, shares etc. where you might also loose your principal amount. It is common that most of the people fear for investing, knowingly or unknowingly they are conscious about risks associated with it. In other words, it is the reward for any chance that you would not get your money back or that it will have declined in value while invested.

(c) Inflation factor

Purchasing power of money goes down at a constant rate annually and we call it inflation. The money we invested should at least earn to cover the loss in its value due to inflation. In other words, it is the compensation for decrease in purchasing power between the time you invest it and time it is returned to you.

Every investor, because of these factors, looks for some return on their investment and charges a cost of investment known as *interest rate*. It is a percentage that is periodically applied and added to an amount (or varying amounts) of money over a specified length of time. When money is borrowed, the interest paid is the charge to the borrower for the use of the lender's property; when money is loaned or invested, the interest earned is the lender's gain from providing a good to another. Interest, then may be defined as the cost of having money available for use.

Elements of transaction involving interest

1. An initial amount of money that, in transaction involving debt or investment, is called the **Principal.**

2. The **interest rate** that measures the cost or price of money and that is expressed as a percentage per period of time.
3. A period of time, called **interest period** that determines how frequently interest is calculated.
4. A specified length of time that marks the duration of the transaction and thereby establishes a certain **number of interest periods**.
5. A future amount of money that results from the cumulative effects of the interest rate over a number of interest periods.

Simple Interest: It uses fixed percentage of the principal (the amount of money borrowed) i.e. if the total amount of interest earned is directly proportional to the initial principal amount, then the interest is said to be simple.

For a deposit of P dollars at a simple interest rate of i for N periods, the total earned interest I would be $I = (i*P) N$

The total amount available at the end of N periods, F, thus would be

$$F = P + I = P\,(1 + iN)$$

Compound interest: When the total time period is subdivided into several interest periods (one year, half yearly, quarterly, monthly, weekly); interest is credited at the end of each interest period, and is allowed to accumulate from one interest period to next, then the interest is said to compounded.

3.2 Derivation of compound interest formula (single cash flow)

(To find the single future sum (F) of the initial payment P)

During a given interest period, the current interest is determined as a percentage of the total amount owed (i.e. principal plus the previous accumulated interest)

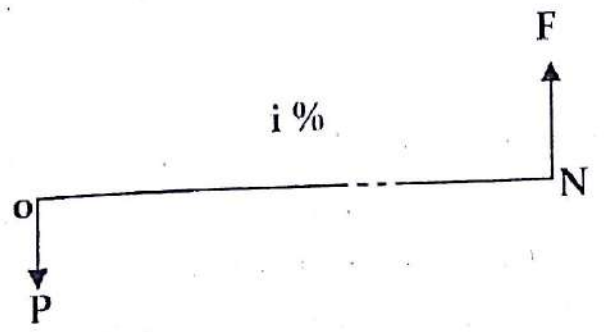


Fig: 3.1Single cash flow diagram

For the 1st interest period,

Interest $I_1 = i*P$

Total accumulated amount at the end of 1st year

$F_1 = P + I_1 = P + i*P = P(1+i)$

For the 2nd interest period

Interest $I_2 = i*F_1 = i*(1+i)$ Total accumulated amount at the end of 2nd year

$F_2 = F_1 + I_2 = P(1+i) + i*(1+i)P = P(1+i)^2$

For the 3rd interest period

$I_3 = F_2 * i = P(1+i)^2 * i$

Total accumulated amount at the end of 2nd year

$F_3 = F_2 + I_3 = P(1+i)^2 + P(1+i)^2 * i = P(1+i)^3$

Continuing,

If there is 'N' interest period

$$F = P(1+i)^N$$

The factor in the bracket is called the ***Single Payment Compound Factor***

Functionally, $F = P(F/P, i\%, N)$

(To find the initial payment (P) of the single future sum (F))

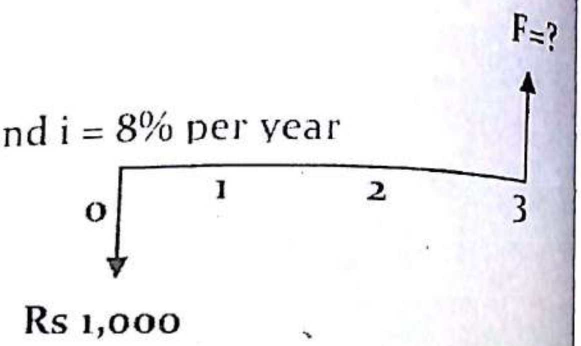
$$P = F(1+i)^{-N}$$

The factor in the bracket is called the ***Single Payment Present worth factor***

Functionally, $P = F\,(P/F, i\%, N)$

Example3.1

Suppose you deposit Rs 1,000 in savings account that pays interest at a rate of 8%, compounded annually. Assume that you don't withdraw the interest earned at the end of each period (year), but let it accumulate. How much would you have at the end of year 3?



Solution

Given: P = Rs 1,000, N=3 years, and i = 8% per year

$F = P\,(F/P, i\%, N)$

$F = \text{Rs } 1{,}000\,(1+0.08)^3$

F = **Rs 1,259.71 (Ans)**

Example3.2

You have just purchased shares @Rs 100 per share. If you expect the stock price to increase 20% per year, how long do you expect to double its market price?

Solution

$F = 2P$

$P\,(1+0.2)^N = 2P$

$1.2^N = 2$

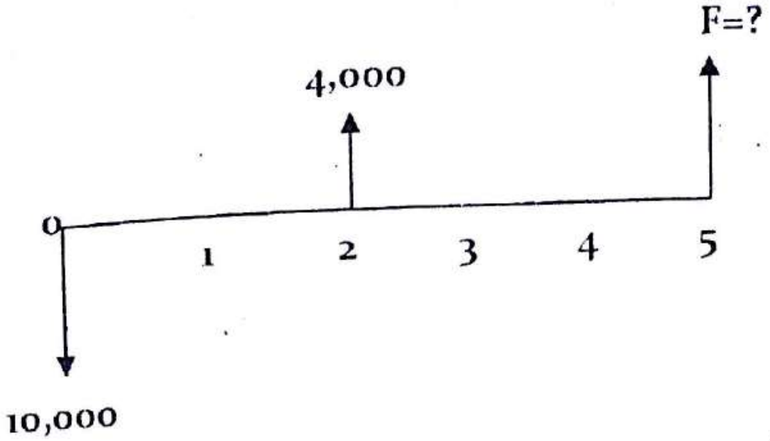
Taking log on both sides

$N \text{ Log } 1.2 = \text{Log } 2$

$N = \text{Log } 2 / \text{Log } 1.2 = 3.80 \approx$ **4 years (Ans)**

Example3.3

Mr. X deposits Rs 10,000 now in a bank which gives 8% interest per year. He draws Rs 4,000 at the end of 2nd year. What will be the remaining amount at the end of 5th year?



At the end of the 2^{nd} year, the accumulated amount will be

$10,000 (F/P, 8\%, 2) = 10,000 (1+0.08)^2 = $ Rs 11,664

After drawing 4000, the remaining deposit amount at the end of 2^{nd} year will be,

$11,664 - 4,000 =$ Rs 7,664

At the end of the fifth year the total accumulated amount will be

$7,664 (F/P, 8\%, 3) = 7,664 (1+0.08)^3 =$ **Rs 9,654.5 (Ans)**

Alternatively

$F = \{10,000 (1.08)^2 - 4,000\} (1.08)^3$

= **Rs 9,654.5 (Ans)**

RULE OF 72

Rule of 72 can determine approximately how long it will take for a sum of money to 'double'. The rule states that "to find the time it takes for the present sum of money to grow by a factor of 2, we divide 72 by the interest rate"

As in the previous example 3.2, 72/20 = 3.60 or roughly we can say 4 years

3.3 Economic Equivalence

Which option would you prefer?

- Receiving Rs. 20,000 today and Rs. 50,000 ten years from now.

- Receiving Rs. 8,000 each year for the next ten years.

Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence. The time value of money and the interest rate helps to develop the concept of economic equivalence which means that different sums of money at different times are equal in economic value. For example, if the interest rate is 6% per year, Rs 100 today (present time) is equivalent to Rs 106 one year from today. If someone offered you a gift of Rs 100 today or Rs 106 one year from today, it would make no difference which offer you accept. In either case you have Rs 106 one year from today. However, the two sums of money are equivalent to each other only when the interest rate is 6% per year. At a higher or lower interest rate, Rs 100 today is not equivalent to Rs 106 one year from today.

Calculations for determining the economic effects of one or more cash flows are based on the concept of economic equivalence. ***Economic Equivalence*** refers to the fact that a cash flow- whether single payment or a series of payment- can be converted to an equivalent cash flow at any point in time. Economic Equivalence exists between cash flows that have the same economic effect.

Example3.4

Suppose we invest Rs. 1000 at 12% annual interest for 5 years.
From Compound interest formula,
$F = P(1+i)^N$, expresses the equivalence between present amount, P, and future amount, F, for given interest, i, and number of interest periods N.
$F = \text{Rs. } 1000(1+0.12)^5 = \text{Rs. } 1762.34$
We can say that at 12% interest, Rs. 1000 received now is equivalent to Rs. 1762.34 received in 5 years, and that we can

trade Rs. 1000 now for the promise of receiving Rs. 1762.34 in 5 years.

Example3.5

Suppose you are offered the alternative of receiving either Rs. 3000 at the end of 5 years or P Rs today. There is no question that the Rs. 3000 will be paid in full (no risk). Because you have no current need for the money, you would deposit the P Rs in an account that pays 8% interest. What value of P would make you indifferent to your choice between P Rs today and the promise of Rs. 3000 at the end of 5 years?

Solution

Our job is to determine the present amount that is economically equivalent to $3000 in 5 years given the investment potential of 8% per year.

Given: F = Rs. 3000, N = 5 years, i=8% per year.

$P = F(1+i)^{-N} = 3000(1+0.08)^{-5}$ = Rs. 2042

- If P is anything less than Rs. 2042, you would prefer the promise of Rs. 3000 in 5years to P Rs today.
- If P were greater than Rs. 2042, you would prefer P.

Equivalence calculation: General Principles

Principle 1

Equivalence calculations made to compare alternatives requires a common time basis.

- In example 3.5, if we had been given magnitude of each cash flow and had been asked to determine their equivalency, we should choose the reference point and find the value of each cash flow at that point.
- For selecting the reference point, commonly present time (present worth) or some point in future (future worth) is used.
- The choice of point is chosen as per convenience.

Example3.6

In example 3.5, we determined that, given an interest rate of 8% per year, receiving Rs. 2042 today is equivalent to receiving Rs. 3000 in 5 years. Are these cash flows also equivalent at the end of year 3?

Solution

Here base period is 3 years.

In fig (a)

P = Rs. 2042, i=8% per year, N = 3 years

In fig (b)

F = Rs. 3000, i=8% per year, N = 2 years

Figure (a)

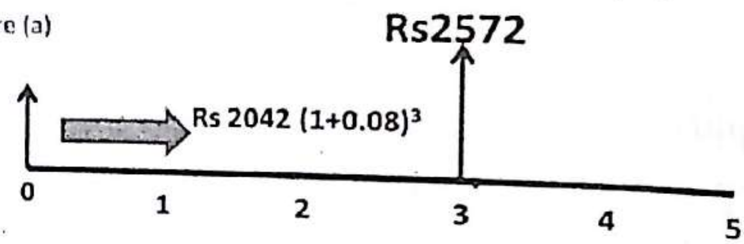
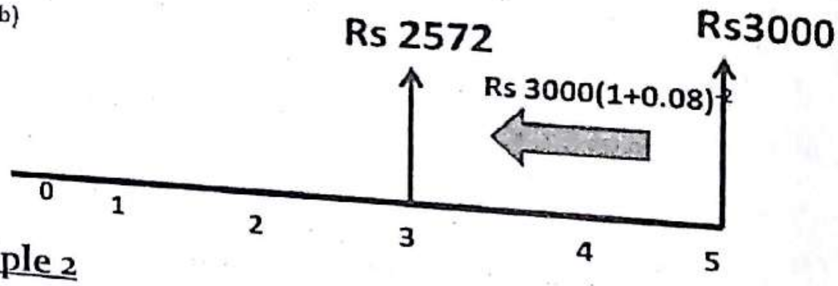


Figure (b)



Principle 2

Equivalence depends on interest rate

- The equivalence between two cash flows is a function of both the cash flow pattern and the interest rate that operates on those cash flows.
- Change in the interest rate will destroy the equivalence between these two sums.

Example3.6

In example 3.5, we determined, at an interest rate of 8% per year, receiving Rs. 2042 today is equivalent to receiving Rs. 3000 in 5 years. Are these cash flows equivalent at an interest rate of 10%?

Solution

Lets select base period N= 5 years.

F = Rs. 2042 (1+0.1)5 = Rs. 3829

Since the amount is > Rs. 3000, change in interest rate destroys the equivalence between two cash flows.

Principle 3

Equivalence Calculations may require the conversion of multiple payment cash flows to a single cash flow.

Example3.7

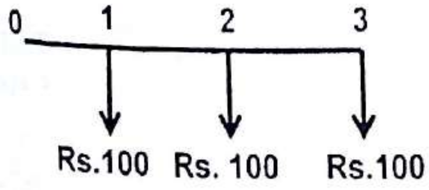
Suppose that you borrow Rs. 1000 from a bank for 3 years at 10% annual interest. The bank offers two options: (1) Repaying the interest charges for each year at the end of that year and repaying the principal at the end of 3 year. (2) Repaying the loan all at once (including both interest and principal) at the end of year 3.

Option	Year 1	Year 2	Year 3
End of year repayment of interest and principal repayment at end of loan.	Rs. 100	Rs.100	Rs.1100
One end of loan repayment of both principal and interest	0	0	Rs. 1331

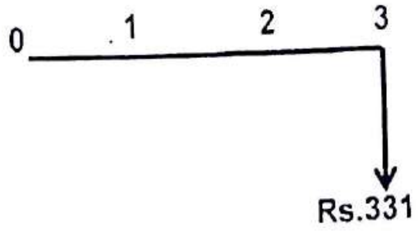
Determine whether these options are equivalent, assuming the interest rate for comparison is 10%?

Solution

Since we pay the principal after 3 years in either plan, the repayment of principal can be removed from our analysis.



Option 1



Option 2

Since option 2 is already a single payment at n=3 years, it is simplest to convert option 1 cash flow pattern to a single value at n=3.

We must convert the three disbursement of option 1 to their respective values at n=3.

F_3 for Rs. 100 at n=1: Rs. 100 $(1+0.10)^{3-1}$ = Rs. 121

F_2 for Rs. 100 at n=2: Rs. 100 $(1+0.10)^{3-2}$ = Rs. 110

F_1 for Rs. 100 at n=3: Rs. 100 $(1+0.10)^{3-3}$ = Rs. 100

Total = Rs. 331

3.4 Nominal and Effective interest rate

If a financial institution uses a unit of time other than a year - a month or a quarter (e.g. when calculating interest payments), the institution usually quotes the interest rate on an annual basis. Commonly this rate is stated as

r% Compounded M-ly

Where, r = the nominal interest rate per year

M = the compounding frequency or the number of interest periods per year

r/M = the interest rate per compounding period.

Suppose that if a bank express the interest rate as "18% compounded monthly", we say that 18% is the *nominal interest rate or annual percentage rate (APR)* and the compounding frequency is monthly i.e. number of interest period per year is 12.

A nominal interest rate *r* may be stated for any time period - 1 year, 6 months, quarter, month, week day etc. Let us see the following examples for expressing the nominal interest rate.

r = 12% compounded semiannually, M=2, i.e.12%/2 (6% per 6 months)

r = 12% compounded quarterly, M=4, i.e. 12%/4 (3% per 3 months)

r = 18% compounded monthly, M=12, i.e. 18%/12 (1.5% per month)

r = 15%compunded weekly, M= 52
i.e.15%/52 (0.29% per week)

When M $\rightarrow \infty$, compounded continuously

Suppose that Rs 1,000 to be invested at a nominal rate of 12% compounded semiannually.

The interest earned during first six months is 1,000* 0.12/2

= Rs 60

Total principal at the end of the first six months = Rs (1,000+60) = Rs 1,060

Interest earned during the second six months is Rs 1,060*0.12/2

= Rs 63.60

Total interest at the end of 1 year = Rs 60+ Rs 63.60

= Rs 123.60

The effective annual interest rate for the entire year = 123.60/1,000 * 100 = 12.36%

The *exact or the actual rate* of interest earned on the principal during one year is known as the ***effective interest (i)***. The effective interest rates are always expressed on an annual basis unless specifically stated otherwise.

Relation between effective (i) and nominal (r) interest rate

$i = (1 + r/M)^M - 1$, M is the compounding period per year.

As from the above example, the effective interest rate for 12% compounded semi annually,

$i = (1 + r/M)^M - 1 = (1+0.12/2)^2 - 1 = 12.36\%$

Alternatively

$i_N = (1+i)^M - 1$,

Where, N = number of compounding periods.

i = interest rate per compounding period

M = compounding periods per year

Example 3.8

What is the effective interest rate of the nominal interest rat 9% per year if the compounding is a) yearly b) quarterly c monthly (d) daily (e) continuously ($N \longrightarrow \infty$)

Solution

For compounding yearly,

$i = (1+0.09/1)^{1} - 1 = 0.09 =$ 9%

For compounding quarterly,

$i = (1+0.09/4)^{4} - 1 = 0.09308 =$ 9.308%

For compounding monthly,

$i = (1+0.09/12)^{12} - 1 = 0.09380 =$ 9.380%

For compounding daily,

$i = (1+0.09/365)^{365} - 1 = 0.0941 = 9.41\%$

For compounding continuously,

$i = (1+0.09/\infty)^{\infty} - 1 = e^{r} - 1 = e^{0.09} - 1 = 9.417\%$

Example 3.9

A person deposits a sum of Rs 5,000 in a bank at a nominal interest rate of 12% for 10 years. The compounding is quarterly. Find the maturity of the deposit after 10 years.

Solution

P = Rs 5,000, N = 10 years, i=12%compounded quarterly

Interest period in a year = 4

Total interest period for 10 years = 40

i= 12%/4 = 3%

Using single payment compound amount factor

$F = P\,(F/P, 3\%, 40) = 5{,}000\,(1+0.03)^{40}$ = **Rs 16,310 (Ans).**

Alternatively

iyear =(1+0.12/4)4 -1 = 12.5508% per year

N =10 years

F = 5,000 (F/P, 12.55%, 10) = 5,000 (1+0.125508)10

= **Rs 16,310 (Ans)**

3.5 Development of Interest formulas

As we begin to compare series of cash flows instead of single payments, the required analysis becomes more complicated. However, when patterns in cash flow transactions can be identified, we can take advantage of these patterns by developing concise expression for computing either present or future worth of the series. We will classify the five major categories of cash flow transactions; develop interest formulas for them and present several working examples for them.

Types of Cash Flow

1. ***Single cash flow:*** the simplest case involves the equivalence of a single present amount and its future worth. The single cash flow formulas deal with the only two amounts; a single present amount P and its future worth F.

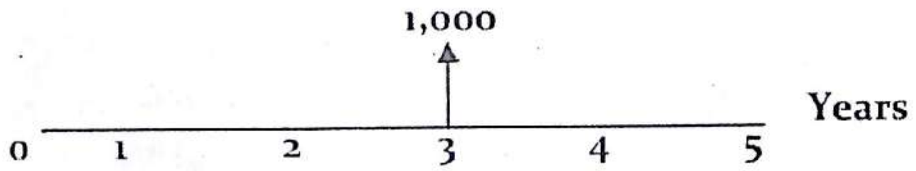


Fig: Single cash flow

2. ***Equal (uniform) series:*** In this type, transactions are arranged as a series of equal cash flows at regular intervals, known as an equal payment series (uniform series) (fig: (b). This describes the cash flows of the common installment loan contract, which arranges repayment of the loan in equal periodic installments.

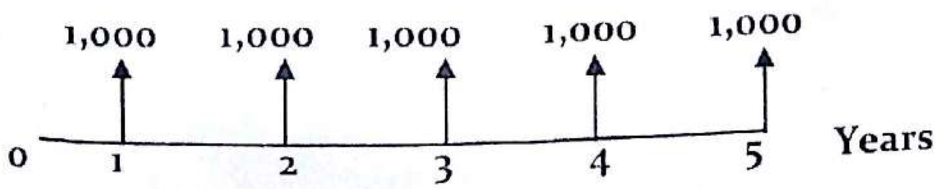


Fig: Equal (uniform) payment series

3. ***Linear gradient series:*** While many transactions involve series of cash flows, the amounts are not always uniform; they may vary in some regular way. One common pattern of variation occurs when each cash flow in a series increases (or decreases) by a fixed amount. For example, A 10 year loan repayment plan might specify a series of annual payments that increase by Rs 1000 each year. This type of cash flow pattern is called linear gradient series.

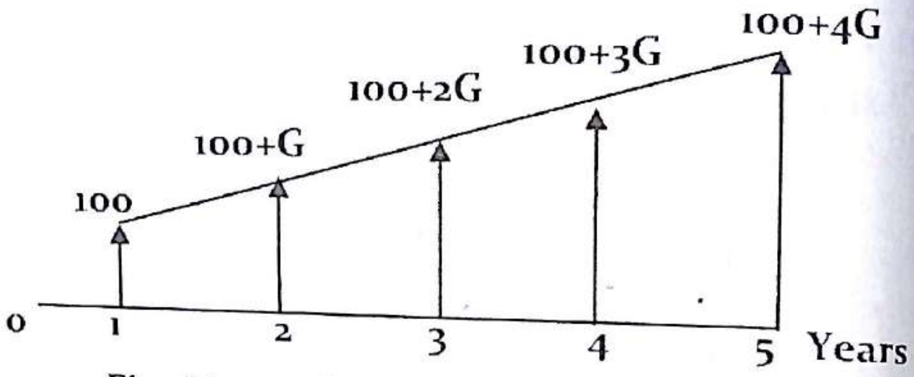


Fig: Linear Gradient series

4. ***Geometric gradient series:*** Another kind of gradient series is formed when the series in cash flow is determined, not by a fixed amount like Rs 1,000, but by some fixed rate, expressed as a percentage. For example, in a 10 year financial plan for a project, the cost of particular raw material might be budgeted to increase at a rate of 4% per year. The curving gradient in the diagram of such a series suggests its name which is geometric gradient series. However, we don't deal with the formulas for geometric gradient series.

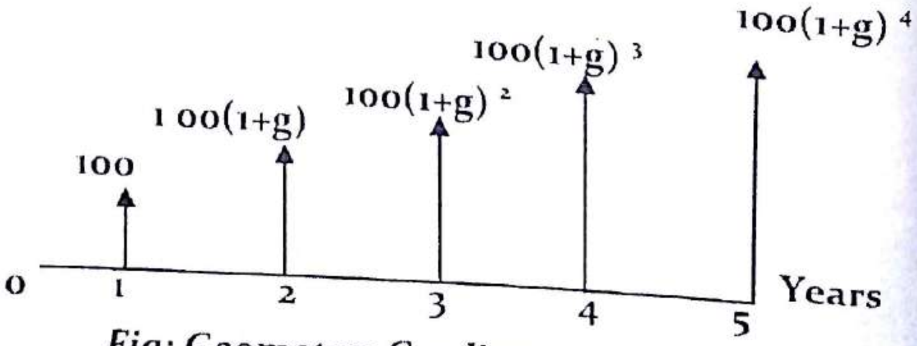


Fig: Geometric Gradient series

5. ***Irregular Series:*** A series of cash flow may be irregular. It doesn't exhibit an overall regular pattern.

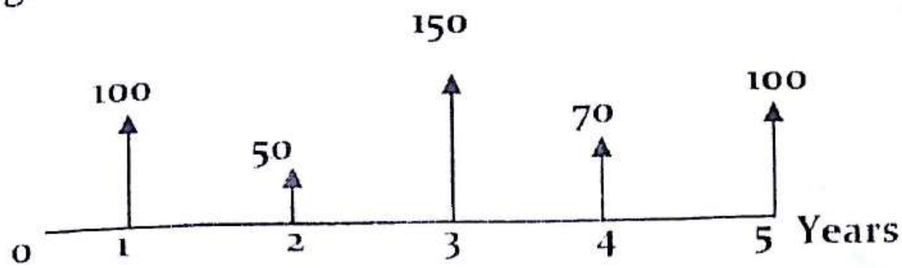


Fig: Irregular series

Factor Notation

We will express the resulting compound interest factors in a conventional notation that can be substituted in a formula to indicate precisely which factor to use in solving an equation. In the preceding examples the formula derived for the single cash flow as $F = P(1+i)^N$. In ordinary language, this tells us that to determine what future amount F is equivalent to a present amount P, we need to multiply P by a factor expressed as 1 plus the interest rate, raised to the power given by the number of interest periods. The factor in the functional form is expressed as: (F/P, i%, N) which is read as "Find F, when P, i, and N given". The factor notation is included for each of the formulas derived in the following sections.

3.6 Discrete compounding and discrete cash flow

Interest formula relating a uniform (equal) series

(1) To Find F when A is given

Let's consider the following cash flow as shown at the end of each period.

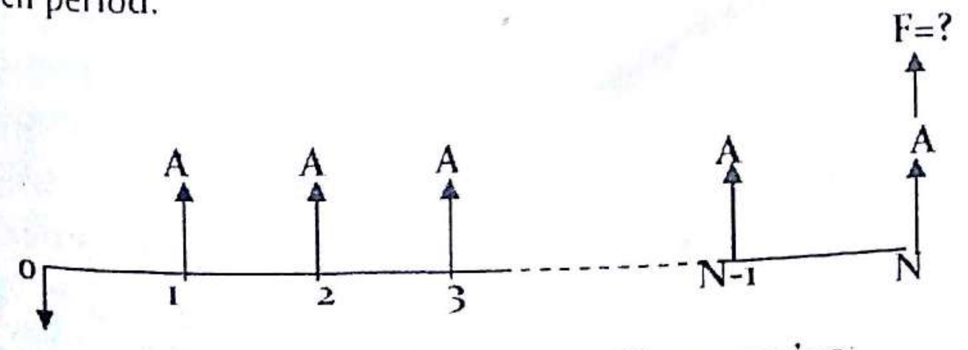


Fig 3.3: cash flow for uniform series

If the amount 'A' occurs at the end of each period for N periods and i% is the interest rate per period, the Future worth (F) at the end of the last (Nth) period is:

$F = A\{(1+i)^{N-1} + (1+i)^{N-2} + (1+i)^{N-3} + \dots\dots\dots\dots + (1+i)^{1} + 1\}$

The bracketed term comprise a geometric progression with common ratio $(1+i)^{-1}$

$$S_N = \frac{a_1 - ba_N}{1-b} \quad (b \neq 1)$$

Where a_1 = first term, b= common ratio, a_N = last term

If we let $b = (1+i)^{-1}$, $a_1 = (1+i)^{N-1}$ and $a_N = 1$, then

$$F = A\left\{(1+i)^{N-1} - (1+i)^{-1} * 1\right\} / \left\{1 - (1+i)^{-1}\right\}$$

$$\mathbf{F = A\{((1+i)^{N} - 1)/i\}}$$

Alternatively,

$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \dots\dots + A(1+i) + A \dots\dots\dots\dots\dots\dots(1)$

Multiplying both sides by (1+i)

$F(1+i) = A(1+i)^{N} + A(1+i)^{N-1} + \dots\dots\dots\dots + A(1+i)^{2} + A(1+i) \dots\dots\dots\dots\dots\dots (2)$

Subtracting equation (1) from equation (2), we get

$F(1+i) - F = A(1+i)^{N} - A$

$F * i = A\{(1+i)^{N} - 1\}$

$$\mathbf{F = A\{((1+i)^{N} - 1)/i\}}$$

The quantity in a bracket is called the *uniform series compound amount factor.*

Functionally, **F = A (F/A, i%, N)**

Note: F (future worth) coincides with last annuity 'A'

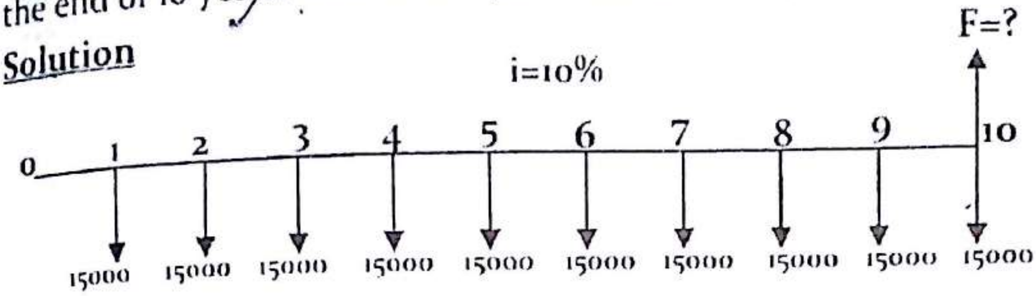
A (annuity) occurs at the end of each period

P (present worth) occurs at on the interest period before the first 'A'

Example 3.10

If a woman deposits Rs 15,000 at the end of each year for 10 continuous period years, how much money is accumulated at the end of 10 years? i=10% compounded annually.

Solution



Using uniform series compound amount factor.

$F = A\ (F/A,\ 10\%,\ 10)$

$F = 15{,}000\ \{((1+0.1)^{10} - 1)/0.1\}$

F = **Rs 2, 39,061.36 (Ans)**

(1) To Find P when A is given

Consider the cash flow diagram in fig: 3.3

We know, $F = A\ [(1+i)^N - 1] / i$

Also, $F = P\ (1+i)^N$

$P\ (1+i)^N = A\ [(1+i)^N - 1] / i$

$$P = A\ [(1+i)^N - 1] / [i * (1+i)^N]$$

The quantity in a bracket is called the *uniform series present worth factor*

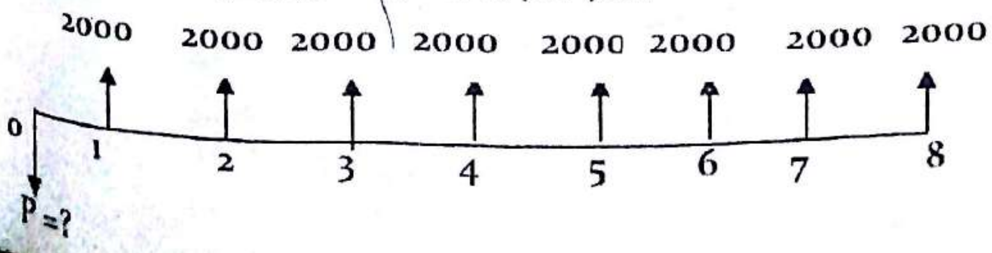
Functionally, $P = A\ (P/A,\ i\%,\ N)$

Example 3.11

How much money should you deposit now in a saving account earning 10% interest rate compounded annually so that you may make 8 end of year withdrawal of Rs 2000 each?

Solution

Given: A = Rs 2,000, N =8, i=10% per year



Using uniform series present worth factor

P= A (P/A, 10%, 8)

$P = 2{,}000\ [(1+0.1)^{8} - 1] / [i * (1+0.1)^{8}]$

P = **Rs. 10, 669.85 (Ans)**

(2) To Find A when F is given

We know, $F = A\ \{[(1+i)^{N} - 1]/i\}$

$$A = F\ [i / ((1+i)^{N} - 1)]$$

The quantity in a bracket is called the *sinking fund factor*

Functionally, A = F (A/F, i%, N)

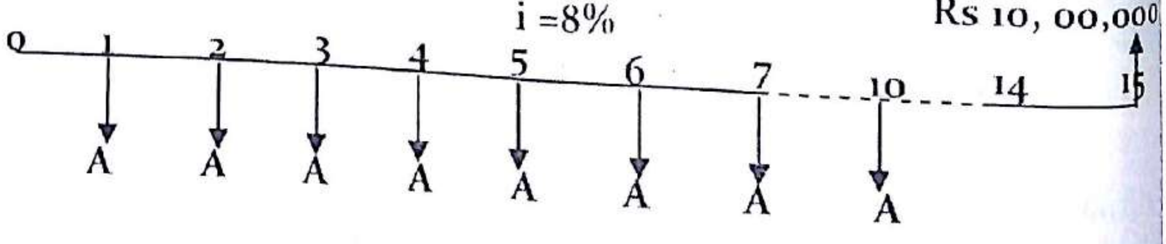
Sinking fund is an interest bearing account into which a fixed sum is deposited each interest period; it is commonly established for the purpose of replacing fixed assets.

Example 3.12

Mr. Ramesh wants to have Rs 10, 00,000 for the studies of his daughter after the period of 15 years. How much rupees does he has to deposit each year for 10 continuous years in a saving account that earns 8% interest annually. **(TU, IOE-2061)**

Solution

Given: F=Rs 10, 00,000, N =15, i=8% per year, A =?



Discounting Rs. 10, 00,000 to the year 10

P = 10, 00,000 (P/F, 8%, 5)

$= 10{,}00{,}000\ (1+0.08)^{-5}$ = Rs. 6, 80,583.197

Using the sinking fund factor

$A = F\ (A/F, 8\%, 10)$

$A = 6,80{,}583.197\ [i / ((1+i)^{N} - 1)]$

$A = 6,80{,}583.197\ [0.08 / ((1.08)^{10} - 1)]$

A = **Rs 46,980.31 (Ans)**

Example 3.13

How many deposits of Rs 25,000 each should Dr. Thakur make each month so that the final accumulated amount will be Rs 10, 00,000 if the bank interest rate is 12% per year? **(TU, IOE 2063)**

Solution

Given: A = 25,000 per month, F = 10, 00,000, i =12% per year

Monthly interest rate,

$i_m = (1+i_{year})^{1/12} - 1$

$i_m = (1+0.12)^{1/12} - 1$

$i_m = 0.0094 = 0.94\%$

Using the uniform series compound amount factor

$F = A\ (F/A, 0.94\%, N)$

$10,00{,}000 = 25{,}000\ \{((1+0.0094)^{N} - 1)/ 0.0094\}$

$0.376 = (1.0094)^{N} - 1$

$1.376 = (1.0094)^{N}$

Taking log on both sides

$\text{Log } 1.376 = N \text{ Log } 1.0094$

N = 34, Dr. Thakur should make 34 **deposits (Ans)**

(1) To Find A when P is given

We know, $P = A\ [(1+i)^{N} - 1] / [i * (1+i)^{N}]$

$$A = P\ [i^*(1+i)^{N}] / [(1+i)^{N} - 1]$$

The quantity in a bracket is called the *capital recovery factor*

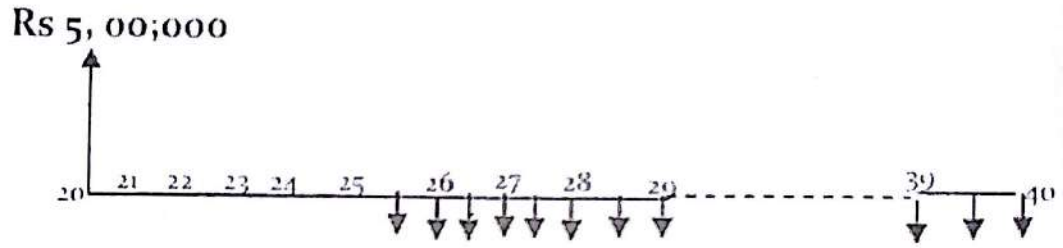
Functionally, **A= P (A/P, i%, N)**

Capital recovery is the annual equivalent of capital cost

Example 3.14

A man aged 40 years now had borrowed Rs, 5, 00,000 from a bank for his further studies at the age of 20 years. Interest was charged at 11% per year compounded quarterly. He wished to pay loan in semiannual equal installments with the first installment beginning 5 years after receiving the loan. He has just cleared the loan now. What amount did he pay in each installment? (TU, IOE-2062)

Solution



Given: P = Rs 5, 00,000, i=11%per compounded quarterly, N=20 years, A =?

Quarterly interest rate,

i_q = 11%/4 = 2.75%

Semiannual interest rate

$i_{semi} = (1+i_q)^2 - 1$

$i_{semi} = (1+0.0275)^2 - 1$

i_{semi} = 5.57%

Using the single payment compound amount factor

F =5, 00,000 (F/P, 5.57%, 40) .... (1)

Using the uniform series compound amount factor

F = A (F/A, 5.57%, 30) .... (2)

Equating equation (1) and (2)

$5,00,000 (1+0.057)^{40} = A \{((1+0.057)^{30} - 1)/ 0.057\}$

A = Rs 61,217.3 is the semiannual payment (Ans)

Interest factor and symbols

To find	Given	Factor	Factor name	Functional symbol
SINGLE CASHFLOW				

F	P	$(1+i)^N$	Single payment compound amount	{F/P, i%, N}
P	F	$1/(1+i)^N$	Single payment present equivalent	{P/F, i%, N}
UNIFORM SERIES				
F	A	$(1+i)^N -1/ i$	Uniform series compound amount	{F/A, i%, N}
P	A	$\frac{(1+i)^N-1}{i*(1+i)^N}$	Uniform Series present worth	{P/A, i%, N}
A	F	$\frac{i}{(1+i)^N-1}$	Sinking fund	{A/F, i%, N}
A	P	$\frac{i*(1+i)^N}{(1+i)^N-1}$	Capital recovery	{A/P, r%, N}

3.7 Continuous compounding and continuous compounding formulas

(a) Continuous compounding and discrete cash flow

The interest formula (for this) assumes, cash whenever it is available, can usually be used profitably. Here cash flows occurs at discrete interval (e.g. once per year) but it is compounded continuously throughout the interval.

Let the nominal rate of interest per year be r, if the interest is compounded continuously for M times and if the principal amount is Rs 1 then the amount at the end of year will be

$1\,(1+r/M)^{M}$ 0.05651.

$= (1+r/M)^{M} -1$ .... (1)

Let $M/r = p$, $M= pr$

Equation one becomes $(1+1/p)^{pr} = \{(1+1/p)^{p}\}^{r}$

Here as $M \rightarrow \infty$, $p \rightarrow \infty$

As $\lim_{P \rightarrow \infty} (1+1/p)^{p} = e$ i =

$\lim_{M \rightarrow \infty} (1+(1/p))^{pr} = e^{r}-1$

$i = e^r - 1$

e^r is equal to (1+i) (i= effective interest rate)

$e^r = (1+i)$

$i = e^r - 1$

We have $F = P(1+i)^N$

$$\mathbf{F = pe^{rN}}$$

$F = P\{F/P, r\%, N\}$

(Continuously compounded compound amount factor for single cash flow)

r% is used to denote the nominal rate and the use of continuous compounding

Interest factors and symbols

To find	Given	Factor	Factor name	Functional symbol
		SINGLE CASHFLOW		
F	P	e^{rN}	Continuous compounding compound amount	{F/P, r%, N}
P	F	e^{-rN}	Continuous compounding present equivalent	{P/F,r%, N}
		UNIFORM SERIES		
F	A	$\frac{e^{rN}-1}{e^r-1}$	Continuous compounding compound amount	{F/A, r%, N}
P	A	$\frac{e^{rN}-1}{e^{rN}(e^r-1)}$	Continuous compounding present equivalent	{P/A, r%, N}
A	F	$\frac{e^r-1}{e^{rN}-1}$	Continuous compounding sinking fund	{A/F, r%, N}
A	P	$\frac{e^{rN}(e^r-1)}{e^{rN}-1}$	Continuous compounding capital recovery	{A/P, r%, N}

(b) ***Continuous compounding and continuous cash flow***

Continuous cash flow means a series of cash flows occurring at infinitesimally short interval. The interest formula could be applied to companies having receipts and expenses that occur frequently during each working day and interest is compounded continuously.

Let, Nominal interest per year = r

If there are p numbers of payment per year which amount to a total of one unit per year, then

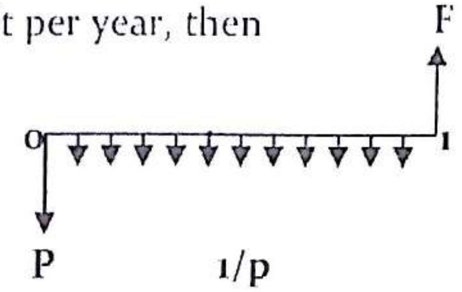


Fig 3.4 Cash flow of Continuous compounding for 1 year

We have

$F = A\{((1+i)^{N}-1)/i\}$

$F = A\{((1+i)-1)/i\}$ (if N=1)

Future equivalent at the end of year 1

$F = 1/p\,\{((1+r/p)^{p}-1)/(r/p)\}$

$F = \{((1+r/p)^{p}-1)/r\}$

As $F = P(1+r/p)^{p}$

$$P = \frac{(1+r/p)^{p}-1}{r(1+r/p)^{p}}$$

As $\lim_{p\to\infty} (1+r/p)^{p} = e^{r}$

Present equivalent of continuous one year cash flow

$P = 1\{e^{r}-1 / r(e^{r})\}$

$P = e^{r}-1/re^{r}$

$(P/A, r\%, N) = er^{N}-1/re^{rN}$

Where A = amount flowing uniformly and continuously over one year.

Interest factors and symbols

To find	Given	Factor	Factor name	Functional symbol

F	A	$e^{rN}-1/r$	Continuous compounding compound amount(continuous uniform cash flow)	{F/A, r%, N}
P	A	$e^{rN}-1/re^{rN}$	Continuous compounding present equivalent(continuous uniform cash flow)	{P/A, r%, N}
A	F	$r/e^{rN}-1$	Continuous compounding sinking fund (continuous uniform cash flow)	{A/F, r%, N}
A	P	$re^{rN}/e^{rN}-1$	Continuous compounding present equivalent	{A/P, r%, N}

F	A	$e^{rN}-1/r$	Continuous compounding compound amount(continuous uniform cash flow)	{F/A, r%, N}
P	A	$e^{rN}-1/re^{rN}$	Continuous compounding present equivalent(continuous uniform cash flow)	{P/A, r%, N}
A	F	$r/e^{rN}-1$	Continuous compounding sinking fund (continuous uniform cash flow)	{A/F, r%, N}
A	P	$re^{rN}/e^{rN}-1$	Continuous compounding present equivalent	{A/P, r%, N}

Example 3.15

Suppose that Mr. Keshab has a present loan of Rs 1,000 and desired to determine what equivalent uniform end of year payments, A, could be obtained from it for 10 years if the nominal interest rate is 20% compounded continuously.

Solution

Here P = Rs 1,000, N=10 years, i=20%compounding continuously, A=?

We know that,

$A = P\ (A/P, r\%, N)$

$= 1{,}000\ \{er^{N}(e^{r}-1)/e^{rN}-1\}$

$= 1{,}000\ \{e^{0.2*10}\ (e^{0.2}-1)/e^{0.2*10}-1\}$

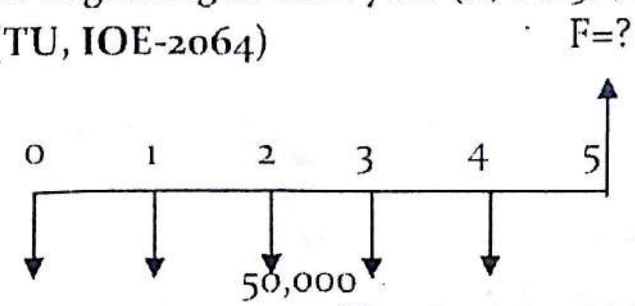
$= 1{,}000\ \{7.389\ (0.221)/6.389\}$

= Rs 256 (Ans)

Example 3.16

Calculate the future worth of the following cash flows deposited at 8% compounded continuously for five years (i) Rs 50,000 at the beginning of each year (ii) Rs 50,000 at the end of each year. **(TU, IOE-2064)**

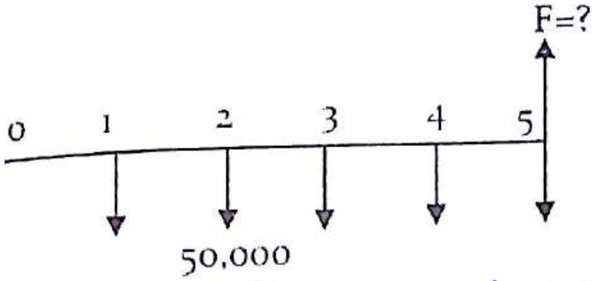
Solution



Using Continuous compounding compound amount factor

$F = A\{F/A, r\%, N\}*e^r = A\,(e^{rN}-1/\,e^r-1))$

$= 50{,}000\,(e^{0.08*5}-1/e^{0.08}-1) * e^{0.08} =$ **Rs 3, 19,850 (Ans)**



Using continuous compounding compound amount factor

$F = A\{F/A, r\%, N) = 50{,}000\,(e^{rN}-1/\,e^r-1)$

$= 50{,}000(e^{0.08*5}-1/\,e^{0.08}-1) =$ **Rs 2, 95,258 (Ans)**

3.8 Interest calculation for linear gradient series

An arithmetic gradient is a cash flow series that either increases or decreases by a constant amount. The amount of increase or decrease is the gradient (G). Formulas previously developed for uniform cash flow series have earned amounts of equal value. In case of a gradient series, each year end cash flow is different, so new formulas must be derived. The cash flow at the end of year 1 is not part of the gradient series, but is rather a base amount. This is convenient because in actual applications, the base amount is usually larger or smaller than the gradient increase or decrease..

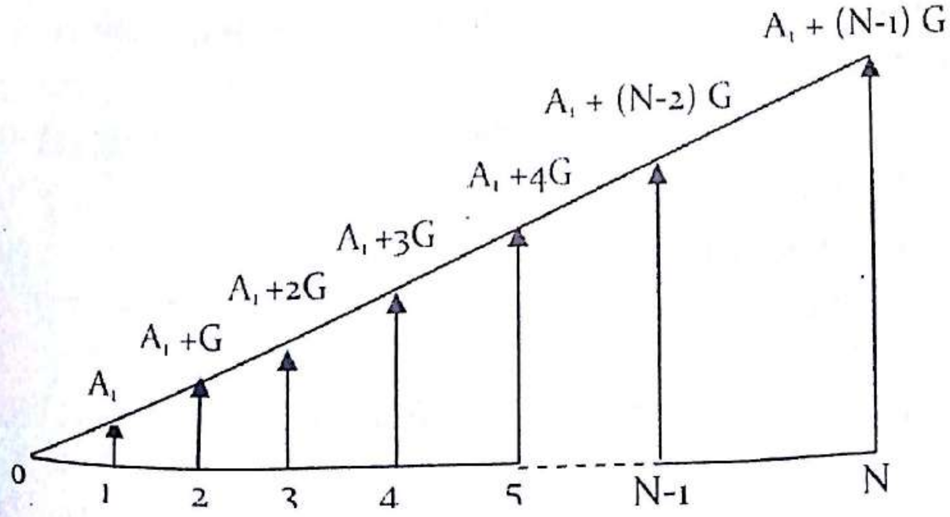


Fig 3.5: Increasing gradient series

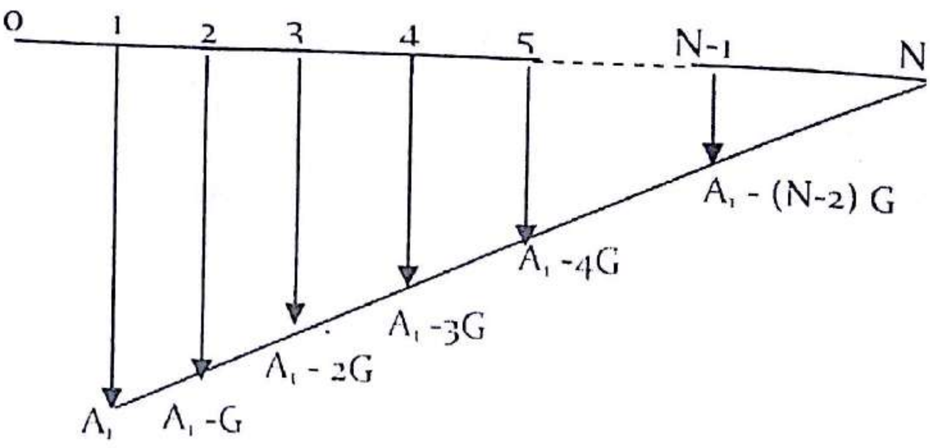


Fig 3.6: Decreasing gradient series

1. To find F when G is given

Let us consider the following cash flow

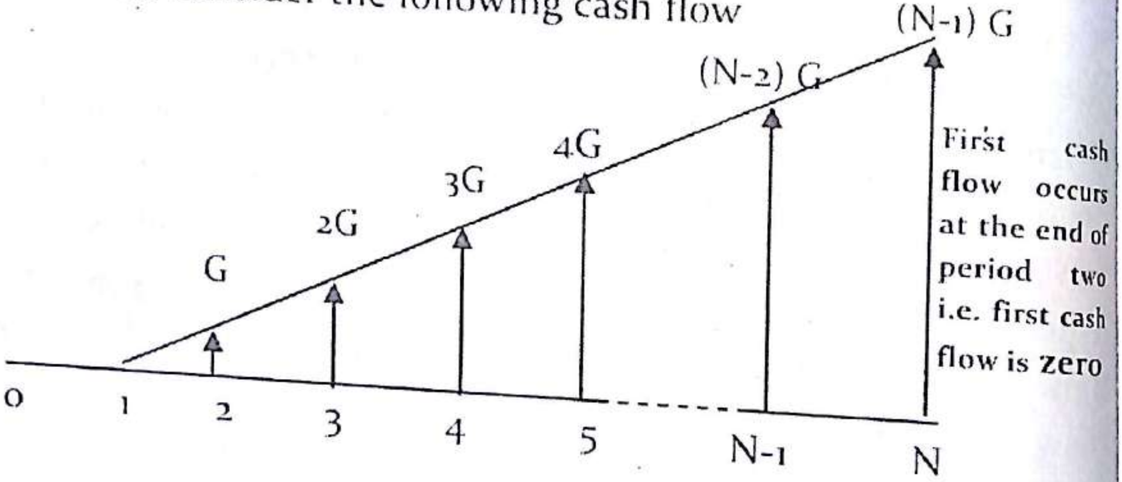


Fig 3.7: Cash Flow diagram for uniform Gradient series

The future equivalent, F, of the arithmetic sequence of cash flows shown in fig 3.6 is

F = G (F/A, i%, N-1) + G (F/A, i%, N-2) + ...+ G (F/A, i%, 2) + G (F/A, i%, 1)

$$F = G \frac{(1+i)^{N-1} - 1}{i} \quad \frac{(i+i)^{N-2} - 1}{i} \quad \frac{(i+i)^{2} - 1}{i} \quad \frac{(i+i)^{1} - 1}{i}$$

$$F = \frac{G}{i} \left[(1+i)^{N-1} + (1+i)^{N-2} + \ldots\ldots + (1+i)^{2} + (1+i)^{1} + 1\right] - \frac{NG}{i}$$

$$F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

The quantity in the bracket is called the *Gradient to Future equivalent factor*

Functionally, $\frac{G}{i}$ (F/G, i%, N) $- \frac{NG}{i}$

2. To find A when G is given

$$F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$A\left[\frac{(i+i)^N - 1}{i}\right] = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$A = G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

The quantity in the bracket is called the *Gradient to uniform series factor*

Functionally, G (A/G, i%, N)

3. To find P when G is given

$$F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$P(1+i)^N = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$\therefore P = \left[\frac{(1+i)^N - 1}{i(1+i)^N}\right]\frac{G}{i} - \frac{NG}{i} * \frac{1}{(1+i)^N}$$

$$P = G\left(\frac{1}{i}\left[\frac{(1+i)^N}{i(1+i)^N}\right] - \frac{N}{i(1+i)^N}\right)$$

$$P = \frac{G}{i^2}\left[\frac{(1+i)^N - Ni - 1}{(1+i)^N}\right]$$

The quantity in the bracket is called the *Gradient to present equivalent factor*

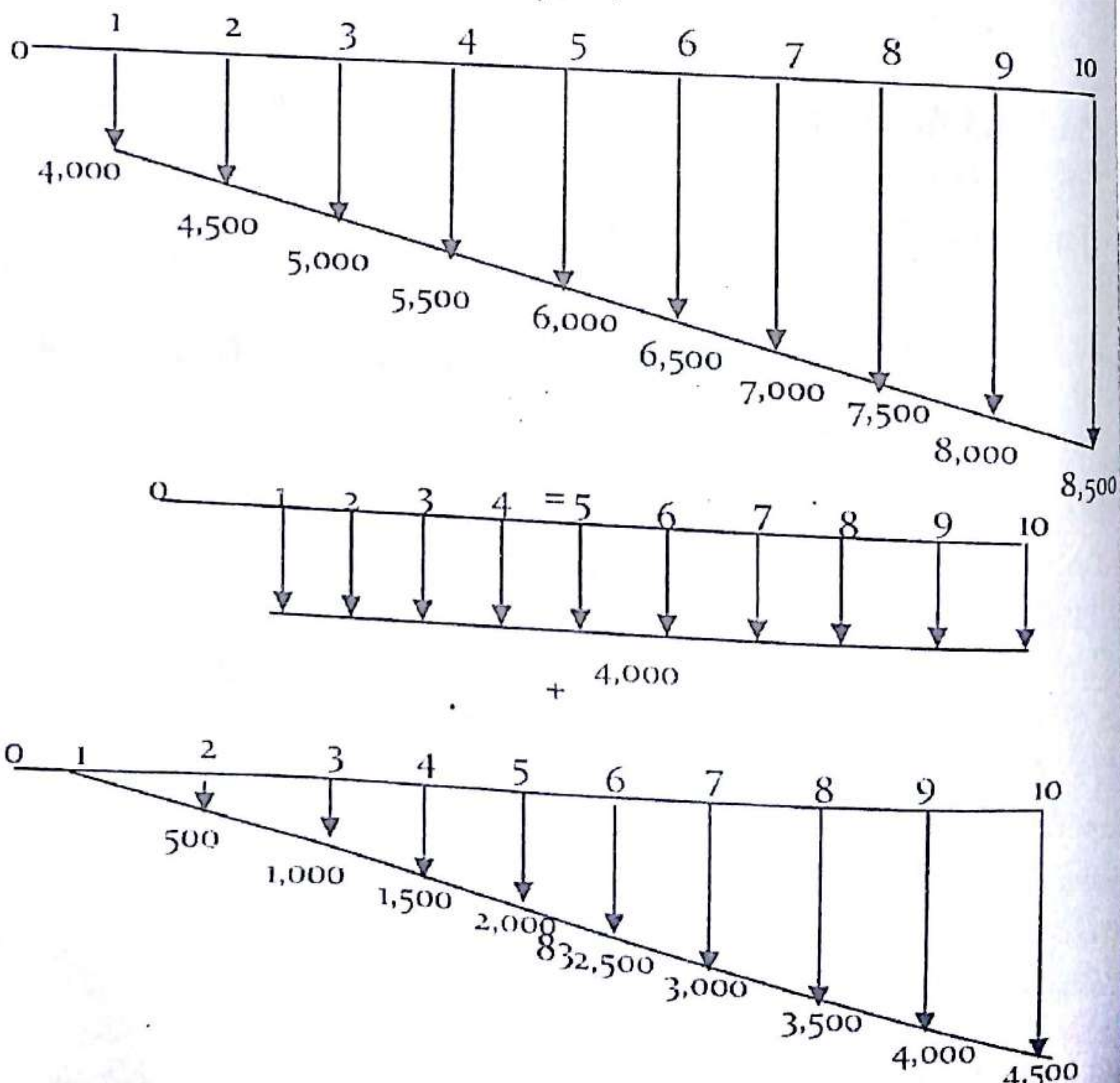
Functionally, G (P/G, i%, N)

Example 3.17

A person is planning for his retired life. He has 10 more years of service. He would like to deposit 20% of his salary, which is Rs 4,000 at the end of the first year, and there after he wishes to deposit the amount with an annual increase of Rs 500 for the next 9 years with an interest rate of 15%. Find the total amount at the end of the 10^{th} year of the above service.

Solution

G = Rs 500, i=15%, N=10 years, F =?



Using uniform series and Gradient to future equivalent factor

$F = F_A + F_G$

$F = A\,(F/A,\ 15\%,\ 10) + G\,(F/G,\ i\%,\ 10)$

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] + \frac{G}{i}\left[\frac{(1+i)^N - 1}{i}\right] - \frac{NG}{i}$$

$$= 4{,}000\left[\frac{(1+0.15)^{10} - 1}{0.15}\right] + \frac{500}{0.15}\left[\frac{(1+0.15)^{10} - 1}{0.15}\right] - \frac{10 * 500}{0.15}$$

= **Rs 1, 15,562 (Ans)**

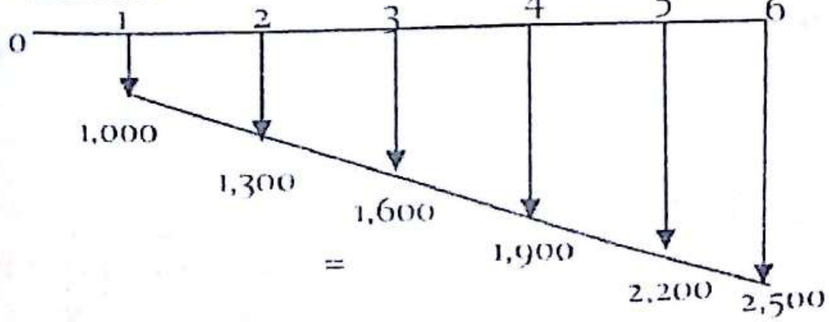
Example 3.18

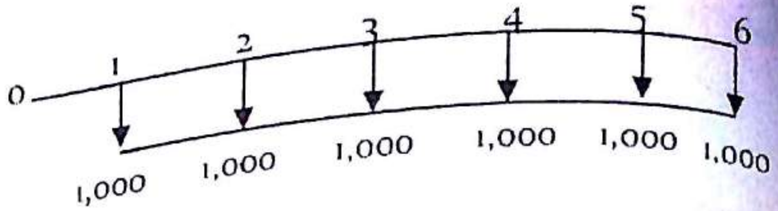
Ram and Shyam have just opened two saving accounts. The accounts earn 10% annual interest. Ram wants to deposit Rs. 1, 000 in his account at the end of the 1st year and increase this amount by Rs.300 for each of the following 5 years. Shyam wants to deposit an equal amount each year for next 6 years. What should be the size of the Shyam's annual deposit so that the two accounts would have equal balance at the end of 6 years?

Solution

Given: G = Rs.300, i=10%, N =6 years

Ram's Cash flow





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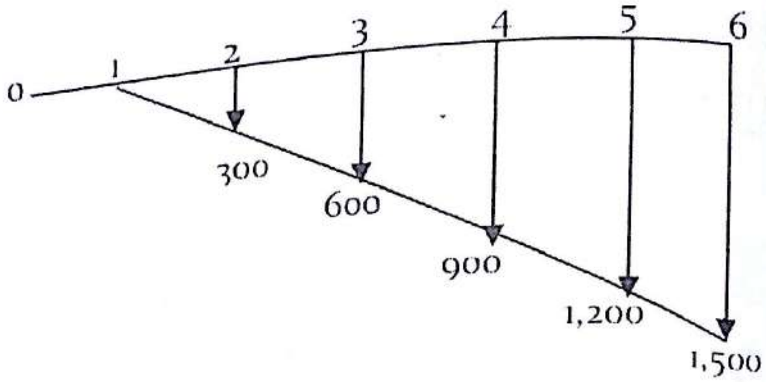


Using Gradient to uniform series conversion factor

A = A +G (A/G, 10%, 6)

$$= 1{,}000 + G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

$$= 1{,}000 + 300\left[\frac{1}{0.1} - \frac{6}{(1+0.1)^6 - 1}\right]$$

= 1,000+ 300(2.2236)

= **Rs. 1,667.08 (Ans)**

Alternatively

P = 1,000(P/A, 10%, 6) + 300(P/G, 10%, 6)

= 1,000 (4.3553) +300 (9.6842)

= Rs.7, 260.56

A = P (A/P, 10%, 6)

= **Rs. 1,667.02 (Ans)** this is Shayam's annual contribution.

Example: 3.19

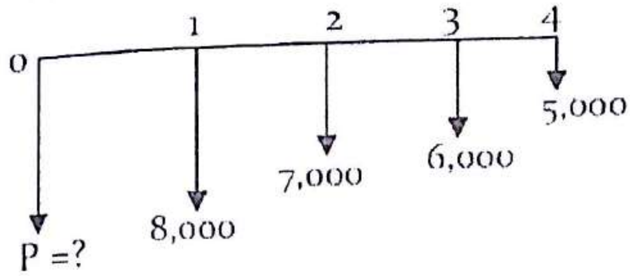
Suppose that one has cash flows as follows.

End of year	Net Cash Flow
1	-8000

2	-7000
3	-6000
4	-5000

Calculate the Present equivalent at i=15%

Solution



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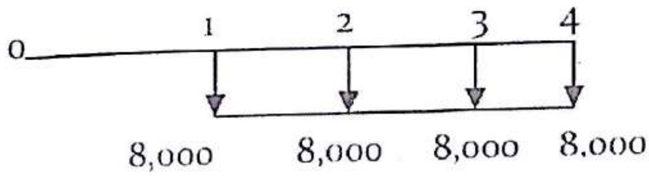

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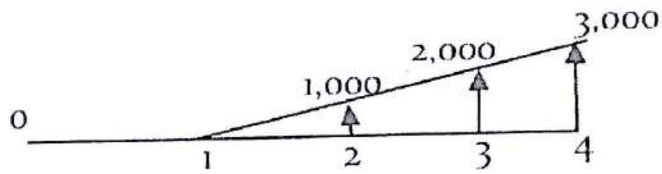


Using Gradient to present equivalent conversion factor

= 8,000 (P/A, 15%, 4) – 1000 (P/G, 15%, 4)

$= 8{,}000\ [((1+0.15)^4 - 1/\ (1+0.15)^4 * 0.15] - 1{,}000/0.15^2$

$$\left[\frac{(1+0.15)^4 - 4*0.15 - 1}{(1+0.15)^4}\right]$$

= 22,864 - 3,786

= Rs 19,078 (Ans)

Example 3.20

Consider the cash flow as below:

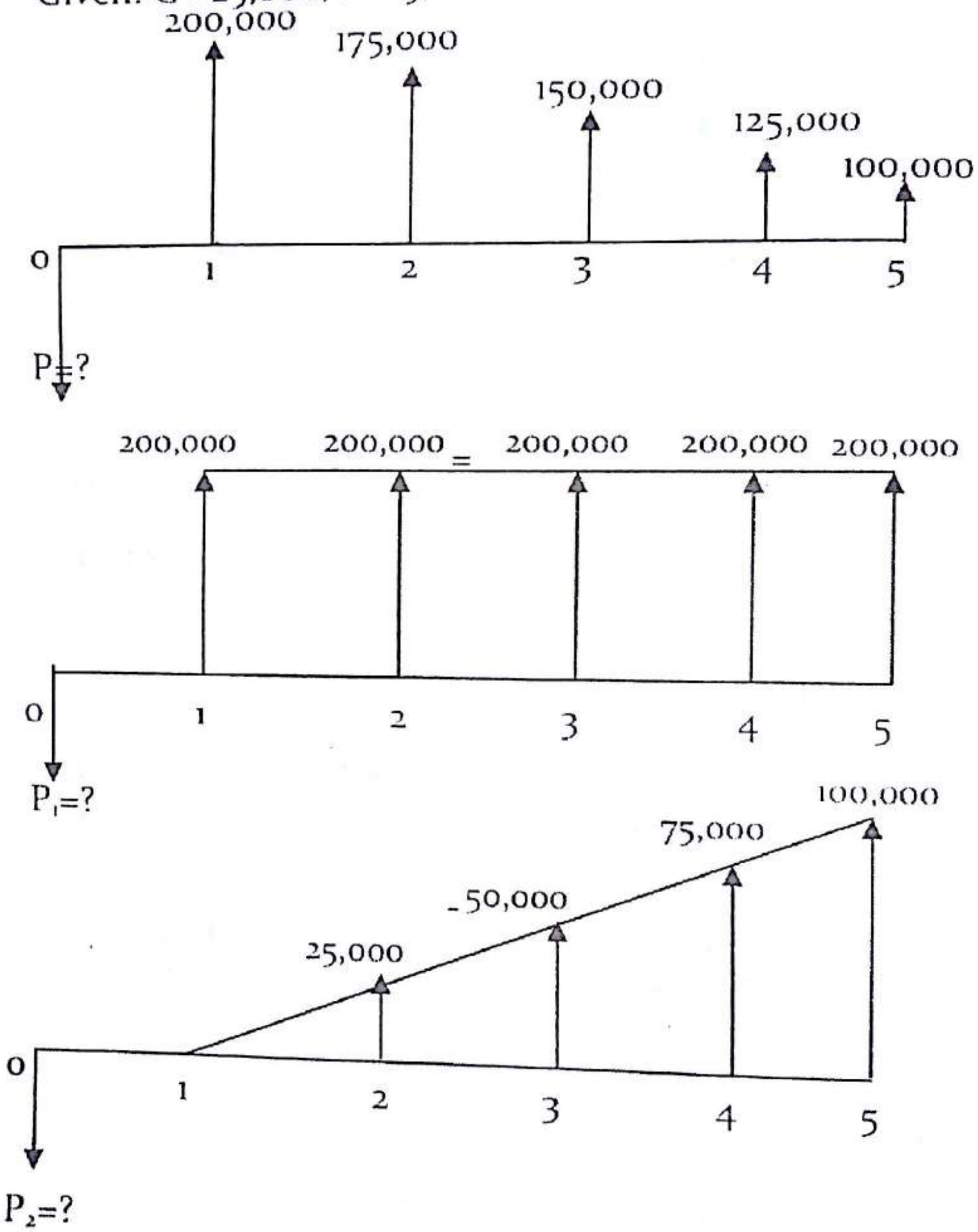
End of year	Net Cash Flow
1	200,000
2	175,000

3	150,000
4	125,000
5	100,000

Calculate the Present equivalent using gradient series formula at i=10% per year.

Solution

Given: G =25,000, N =5, i=10%



Using uniform series present worth factor and Gradient to present equivalent conversion factor

$P = P_1 - P_2$

P = 2, 00,000 (P/A, 10%, 5) – 25,000 (P/G, 10%, 5)

$P = 2,00,000[(1+0.1)^5-1]/[0.1*(1+0.1)^5] - 25,000/(0.1)^2[((1+0.1)^5-5*0.1-1)/(1+0.1)^5$

$P = 7,58,160 - 1,71,545$

$P = 5,86,615$ **(Ans)**

3.9 Interest calculation for geometric gradient series

Many Economic problems involve cash flows that increase or decrease over time, not by a constant amount (linear gradient) but. rather by a constant percentage (geometric), which is called compound growth. We use 'g' to designate the percentage change in payment from one period to the next. The magnitude of n^{th} payment, A_n, is related to the first payment A_1, is expressed by

$A_n = A_1 (1+g)^{n-1}$, **where n=1, 2, 3...N**

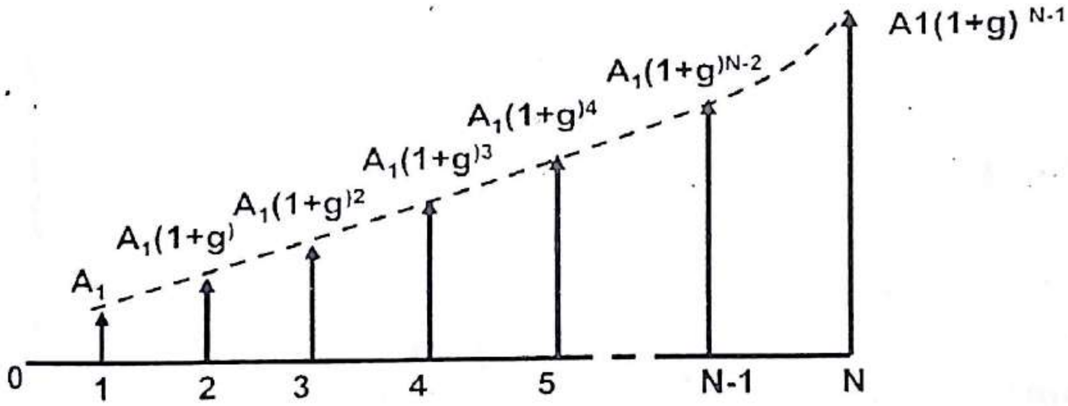


Fig 3.8: Cash Flow diagram for geometric gradient increasing series

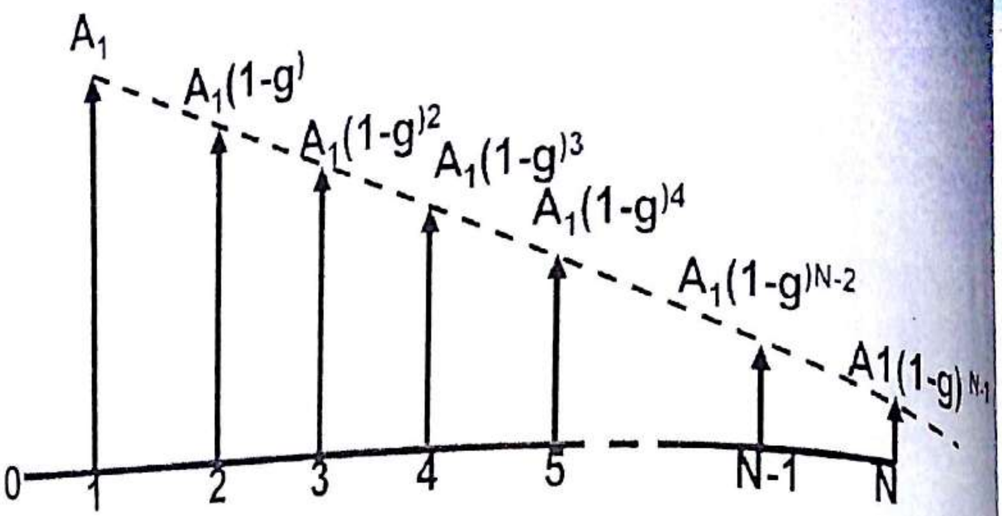


Fig 3.9: Cash Flow diagram for geometric gradient decreasing series

<u>Present worth Factor, Find P, given G, g, i, N</u>

<u>If i ≠ g</u>

$$P = A_1 \frac{[1 - (1+g)^N (1+i)^{-N}]}{i - g}$$

<u>If i = g</u>

$$P = [NA_1 / (1+i)^{-1}]$$

Functionally,

$P = A_1 (P/A_1, g, i, N)$ *Geometric gradient series present worth factor*

<u>Future worth Factor, Find F, given G, g, i, N</u>

<u>If i ≠ g</u>

$$F = A_1 \frac{[(1+i)^{N} - (1+g)^N]}{i - g}$$

<u>If i = g</u>

$$F = NA_1 (1+i)^{N-1}$$

Functionally,

$F = A_1 (F/A_1, g, i, N)$ *Geometric gradient series future worth factor*

Example: 3.21

Air plane ticket price will increase 8% in each of the next four years. The cost at the end of the first year will be Rs. 2000. How much should be put away now to cover a passenger travel home at the end of each year for the next four years. Assume i=5%.

Solution

Given: A_1 =2000, g=8%, i=5%

Here i ≠ g

$$P = A_1 \frac{[1-(1+g)^N (1+i)^{-N}]}{i-g}$$

$$= 2000 \frac{[1-(1+0.08)^4 (1+0.05)^{-1}]}{0.05-0.08}$$

$$= 20000^* 3.9759$$

$$= \textbf{Rs. 7951.84 (Ans)}$$

For Check: Without Using Geometric gradient

For year 1: $A_1 = 2000$

For year 2: $A_2 = 2000 + 0.08\,(2000) = 2160$

For year 3: $A_3 = 2160 + 0.08\,(2160) = 2332.8$

For year 4: $A_3 = 2333.8 + 0.08\,(2332.8) = 2519.24$

P = 2000 (P/F, 5%, 1) + 2160 (P/F, 5%, 2) + 2332.8 (P/F, 5%, 3) + 2519.24 (P/F, 5%, 4)

$= 2000\,(1+0.05)^{-1} + 2160\,(1+0.05)^{-2} + 2332.8\,(P/F, 5\%)^{-3} + 2519.24\,(1+0.05)^{-4}$

= 1904.76 + 1959.18 + 2015.16 + 2072.58 = **7951.684 (Ans)**

Example: 3.22

An investment of Rs. 100,000 is made in a limited partnership in a natural gas drilling project. The first year of the investment produced net revenue of Rs. 25,000. Over a 20 year period, the net revenue received from the investment decreased by 10% each year. Based on a interest rate of 12%, what is the present worth for the investment?

Solution

Given: A_1 = 25,000, i =12%, g= -10%, and N=20 years.

PW = -100,000 + 25,000 (P/A_1, 12%, -10%, 20)

$$= -100,000 + 25,000 \frac{[1-(0.9)^{20}(1.12)^{-20}]}{012-(-0.1)}$$

= 12,204.15 (Ans)

Example: 3.23

A graduating Civil Engineer is going to make Rs 35000 per year with granite construction. A total of 10% of the salary will be placed in the mutual fund of their choice. The engineer can count on a 3% salary increase for the next 30 years of employment. If the engineer place the retirement fund in a account averaging 12% over the course of a career. What can he expect at retirement?

Solution

Given: A_1 = 10% of 35000 = Rs 3500, i=12%, g=3%, N =30 years

Here i ≠ g

$$F = A_1 \frac{[(1+i)^{N} - (1+g)^{N}]}{i-g}$$

$$= 3500 \frac{[(1+0.12)^{30} - (1+g)^{30}]}{0.12-0.03}$$

= Rs 1070714

3.10 Introduction to Financial and Economic Analysis

Financial and economic analysis are required to determine overall project feasibility and sometimes use the same data, they are conceptually different type of analysis. The objective of economic analysis is to determine if a project represents the best use of resources over the analysis period. (That is the project is economically justified). The objective of financial analysis is to determine financial feasibility (that is whether someone is willing to pay for a project and has a capability to raise the necessary funds).

In financial analysis, evaluation is from the perspective of parties expected to pay their allocated costs. Evaluation period is the bond repayment period. Interest paid during construction is included. Project income and capital and annual operation costs are estimated in inflated rupees. In economic analysis, evaluation is from the many perspective (like individuals, communities etc). Evaluation period is the economic life of the project. Project benefits and capital and annual operation costs are estimated in un-inflated rupees. Financing costs are not included.

The main objective of an individual, affirm or a company in investing on a project is to earn the maximum possible returns for the investment. Accordingly, the project promoters are solely interested in wealth maximization. Hence the project promoters tend to evaluate only the commercial (financial) profitability of a project. There are some projects that may not offer attractive returns as far as financial profitability is concerned, but still such projects are undertaken since they have social implications. Such projects are public projects like road, bridge, irrigation project, power project etc for which socio-economic considerations play a significant part rather than mere commercial profitability. Such projects are analyzed for their net socio economic benefits.

As against the market prices of direct cost and direct benefits considered in financial profitability analysis, the economic analysis takes into account the 'real cost' of direct costs and 'real benefit' of direct benefits. For example, some of the inputs (say, power charges) may be subsidized. Only the subsidized prices of inputs are what is relevant for assessing financial profitability. However the cost benefit analysis takes into account the real cost of inputs i.e. the cost inputs had they not been subsidized. Accordingly the required adjustments to direct costs of inputs are made for social cost benefit analysis.

Similarly, cost adjustments may also be required for the benefits. For example, the output may be a product whose price is controlled by the government, the commercial profitability analysis will take into account the controlled price (which is the market price fixed by the government) while its actual benefit to the nation may be more or less than the controlled price, which is what is relevant for social cost benefit analysis.

3.11 Life Cycle Cost

The Life cycle cost, as it name implies is commonly applied to alternatives with cost estimates over the entire system life span. This means that costs from the very early stage of the project (initiation) through the final stage (phase-out and disposals) are estimated. Typical applications for life cycle cost are buildings (new construction or purchase), manufacturing plants, commercial aircraft, new automobiles model and the like. A present worth analysis with all definable costs estimated may be considered a life cycle cost analysis. Life cycle cost is most effectively applied when a substantial percentage of the total costs over the system life span, relative to the initial investment, will be operating and maintenance costs. To understand how a life cycle cost analysis works, first we must understand the phases and stages of system development. Generally, life cycle cost estimates may be categorized into a simplified format for the major phases of acquisition and operation and their respective stages.

Acquisition phase: all activities prior to the delivery of products and services.

- Requirements definition stage: Includes determination of user/ customer needs, assessing them relative to the anticipated system, and preparation of the system requirements documentation.

- Preliminary design stage: Includes feasibility study, conceptual, and early-stage plans; final go/no go decision is probably made here.
- Detailed design stage: Includes detailed plans for resources-capital, human, facilities, information system, marketing etc. there is some acquisition of assets, if economically justifiable.

Operation phase: all activities are functioning, product and services are available.

- Construction and implementation stage: Includes purchase, construction, and implementation of system components, testing, preparations etc.
- Usage stage: Uses the system to generate products or services.
- Phase-out and disposal stage: Covers time of clear transition to new system; removal/recycling of old system.

Some solved examples

1. Mr. X receives a loan of Rs 1, 20,000 from a bank at an interest rate of 12% per year. He wishes to repay the loan in monthly installment with Rs 3,000 per month. How many installments are necessary to complete his payment? (**TU, IOE 2066**)

Solution

Given: P = Rs 1, 20,000, i=12% per year, A = 3,000 per month, N =?

Since i is in year and A is in month, it is not compatible. Find i per month first using

$$i_m = (1+i_{year})^{1/12} - 1$$

$$= (1+0.12)^{1/12} - 1 = 0.95\%$$

Now using the concept of Equivalence

120,000 = 3,000 (P/A, 0.95%, N)

$40 = ((1+0.0095)^N - 1) / 0.0095*(1+0.0095)^N$

$0.38(1+0.0095)^N = (1+0.0095)^N - 1$

$0.62 (1+0.0095)^N = 1$

Taking log on both sides

$N \log 1.0095 = \log (1/0.62)$

N = **50.5 months (Ans)**

2. Suppose that the guardian of a boy decides to make annual deposits into a saving account, with the first deposit begin on the boy's fifth birthday and the last deposit ends on boy's fifteenth birthday. Then starting on the boy's eighteenth birthday, the withdrawal will be Rs 4,000 every year till the boy's twenty first's birthday. If the effective annual interest rate is 8% during this period of time, what are the annual deposits in year five through fifteen? **(PU 2006)**

Solution

i=8%, withdrawal =Rs 4,000 from 18^{th} to 21^{st} year

Discounting the withdrawal series to year 15

{4,000 (P/A, 8%, 4)} (P/F, 8%, 2) = Rs 11,358

Again applying uniform series compound amount factor

11,358 = A (F/A, 8%, 11)

11,358= 16.645 A

A = **Rs. 682.36 (Ans)**

3. What is equal payment series for 12 years that is equivalent to a payment series of Rs 20,000 at the end of first year increasing by 10% per year? Interest rate is 8% per year. **(TU, IOE, 2069)**

Solution

Given: the first payment (A_1) = 20,000, g= 10%, i=8%,N=12 years, annual equivalent (A) = ?

This is the case of increasing geometric gradient series.

Here i is not equal to g, and we calculate the equivalent future of at the end of year 12 for the given series of cash flow.

$$F = A_1 \frac{[(1+i)^{-N} - (1+g)^{N}]}{i - g}$$

$$= 20,000 \frac{\{(1+0.08)^{12} - (1+0.1)^{12}\}}{0.08 - 0.10}$$

$= \text{Rs } 620258.25$

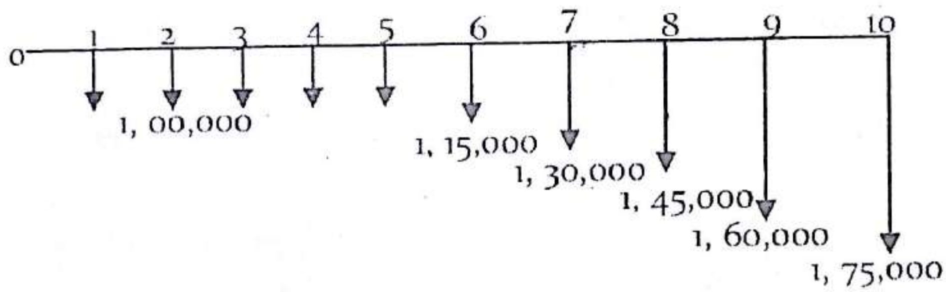
Converting this future value into the annual equivalent

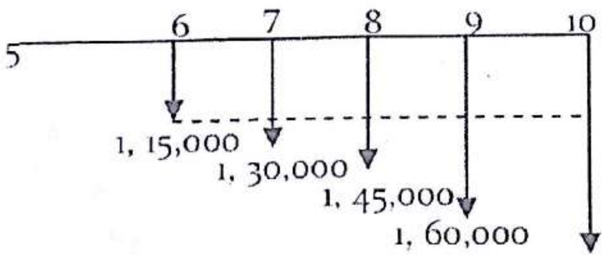
A = 620258.25 (A/F, 10%, 12)

= 620258.25 * 0.0468 = Rs. 29028.08 (Ans)

4. Mr. Kumar has inspected his yearly household expenses for the last 10 years. Cost averages were steady at Rs 1, 00,000 per year for the first 5 years, but have increased consistently by Rs 15,000 per year for each of the last 5 years. Calculate total present worth in year zero. Use gradient formula. **(TU, IOE, 2068)**

Solution





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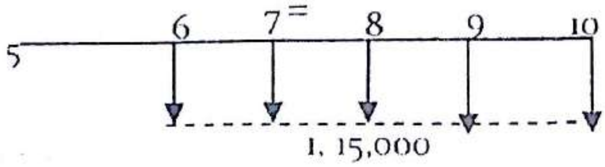

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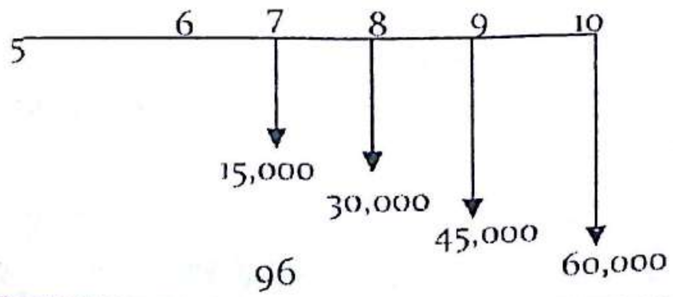


= 115000 (P/A, 10%, 5) + 15000 (P/G, 10%, 5)
= 115000 * 3.7908 + 15000 * 6.8618
= 435942 + 102927
= 538869

Now discounting all the cash flows into year zero.

P = 538869 (P/F, 10%, 5) + 100000 (P/A, 10%, 5)
= 538869 *0.6209 + 100000 * 3.7908
= Rs. 713663.76 (Ans)

Review Questions

1. Explain Time value of Money.
2. What is the practical application of the different interest formulas?
3. Differentiate between nominal and effective interest rate.

Exercises

1. For an interest rate of 8%, compounded annually, find (i) how much can be loaned now if Rs. 5,000 will be repaid at the end of 5 years? (ii) How much will be required in 4 years to repay a Rs 12,000 loan now? **(3402.92, 16325.87)**
2. You bought 100 shares on 2065 Chaitra 31. Your intention is to keep the stock until it doubles in value. If you expect 15% annual growth for shares, how many years do you expect to hold on the stock of shares? Compare the solution obtained by rule of 72. **(5 years)**
3. How much invested now at 6% would be sufficient to provide three payments with the first payment in the amount of Rs 2,000 occurring at the end of 1st year, Rs 3,000 at the end of 5 years, and Rs 4,000 at the end of 3 years? **(7487.04)**

STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- To understand the concept of Present Worth, Future Worth and Annual worth and Capitalized worth Method.
- To understand the concept of Rate of Return such as Internal Rate of Return and External rate of Return
- To perform economic analysis using equivalent worth methods and rate of return methods.
- To evaluate the public sector project using Benefit Cost Analysis Method.
- To understand the concept of Payback Period Method and evaluating the project.

4.1 Introduction

If an organization have a huge sum of money in the investment pool and there are many alternatives (projects) whose initial investment cost and annual revenues are known then the organization has to select the best alternative among the different projects. The worthiness of the each project should be performed. The bases for comparing the worthiness are achieved by the following methods:

1. **Equivalent Worth Method**
   (a) Present worth method
   (b) Future worth method
   (c) Annual worth method
2. **Rate of Return Method**
   (a) Internal rate of return
   (b) External rate of return
3. **Benefit Cost Analysis**
   (a) Conventional
   (b) Modified
4. **Payback Period Method**
   (c) Simple payback period
   (d) Discounted payback period

4.2 Equivalent worth Method

Present worth method (PW)

The Net Present Worth (NPW) or Present Worth or Net Present Value (NPV) of a given series of cash flow is the equivalent values of the cash flows at the end of year zero (i.e. beginning of year 1).In other words, how much money we have to set aside to provide for future cash flow.

Net present worth = Equivalent present worth of future cash flow – Initial investment

Here we use NPW or NPV as PW

Basic procedure

- Determine the interest rate that the firm wishes to earn on their investment, which is referred as either required rate of return or MARR (minimum attractive rate of return).
- Estimate the service life of the project.
- Estimate the cash inflow and out flow for each service period.
- Determine the net cash flows

  = Cash inflow – cash out flow
- Find the Present worth of each net cash flow at MARR. As:-

$$PW(i\%) = \frac{A_0}{(1+i)^0} + \frac{A_1}{(1+i)^1} + \frac{A_2}{(1+i)^2} + \text{--------} + \frac{A_N}{(1+i)^N}$$

$$= \sum_{n=0}^{N} \frac{A_n}{(1+i)^n}$$

$$= \sum_{n=0}^{N} A_n(P/F, i\%, N)$$

- A_n will be +ve if the corresponding period has a net cash inflow and -ve if there is net cash outflow.
- Positive 'NPW' means the equivalent worth of cash inflows is greater than equivalent worth of cash out flows and vice versa

Decision rule

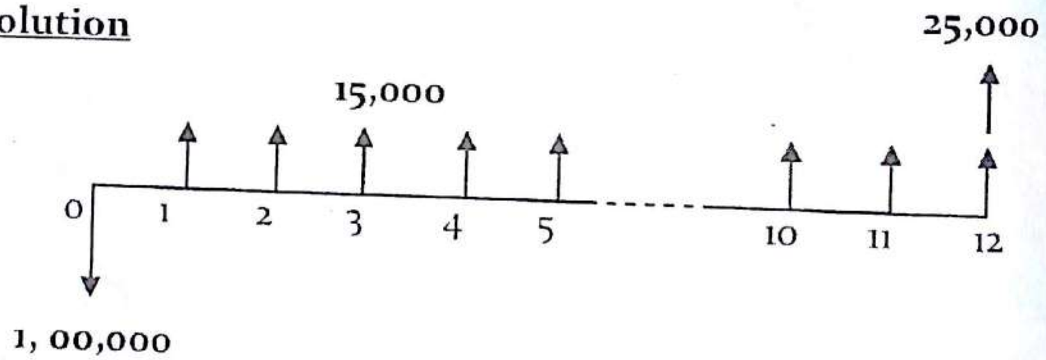
NPW/NPV/PW method gives a definite decision rule. All projects with a positive NPW should be undertaken as these projects will increase the shareholder's wealth as this reflects the present value of future cash flow.

If PW (i) > 0'	*accept the investment*
If PW (i) = 0'	*remain indifferent*
If PW (i) < 0'	*reject the investment*

Example: 4.1

A company intends to mechanize its existing process by installing a machine. The machine is estimated to cost Rs. 1, 00,000 and expected to save Rs. 15,000 per year for a period of 12 years. Is it worthwhile to install the machine if salvage value is Rs. 25,000 at the end of 12 years? MARR =10%. Use PW formulation.

Solution



Using the present worth Formulation

PW (10%) = -1, 00,000 + 15,000 (P/A, 10%, 12) + 25,000 (P/F, 10%, 12)

= -1, 00,000 + 15,000 * 6.8137 + 25,000 * 0.3186

= **Rs. 10,170.5, since PW is positive; it is worthwhile to install the machine.**

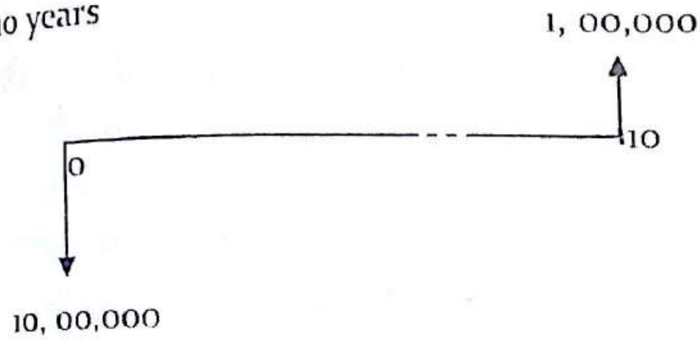
Example 4.2

A construction company needs equipment which costs Rs 10, 00,000 and has a salvage value of Rs 1, 00,000 at the end of 10 years. The equipment suppliers are also willing to provide the equipment on hire for Rs 1, 25,000 per year for 10 years. What will you do? Purchase or Hire. MARR =12% **(TU, IOE-2066)**

Solution

For the purchasing case

Initial cost (P) = Rs 10, 00,000, Salvage value (S) = 1, 00,000, N =10 years



Using PW formulation

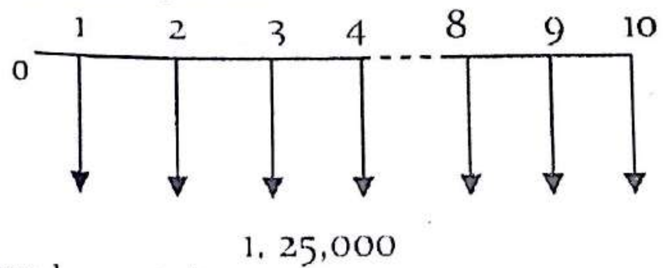
PW (12%) = -10, 00,000 + 1, 00,000 (P/F, 12%, 10)

$= -10, 00,000 + 1, 00,000(1+0.12)^{-10}$

= Rs – 9, 67,802.67

Present worth of purchasing the equipment is Rs. **9, 67,802.67**

For the Hiring case



Annual cost (A) = Rs 1, 25,000

Using PW formulation

PW (12%) = - 1, 25,000 (P/A, 12%, 10)

$$= -1,25,000 \frac{(1+0.12)^{10}-1}{0.12(1+0.12)^{10}}]$$

= Rs -7, 06,277.87

Present worth of hiring the equipment is Rs. **7, 06,277.87**

Since the PW of Hiring cost is less than the PW of the Purchasing cost, **Hire the equipment.**

Example: 4.3

For a proposed manufacturing plant

Land cost:	Rs 3, 00,000
Building cost:	Rs. 6, 00,000
Equipment:	Rs. 2, 50,000
Working capital;	Rs 1, 00,000
Sales revenue:	Rs. 7, 50,000/year for 10 years
Salvage value (after 10 years)	
Land:	Rs. 4, 00,000
Building:	Rs. 3, 50,000
Equipment:	Rs. 50,000
Working capital recovered:	Rs 1, 00,000
Annual expenses:	Rs. 4, 75,000
MARR:	25%

Determine, is the investment on this project is good or bad on the basis of PW method.

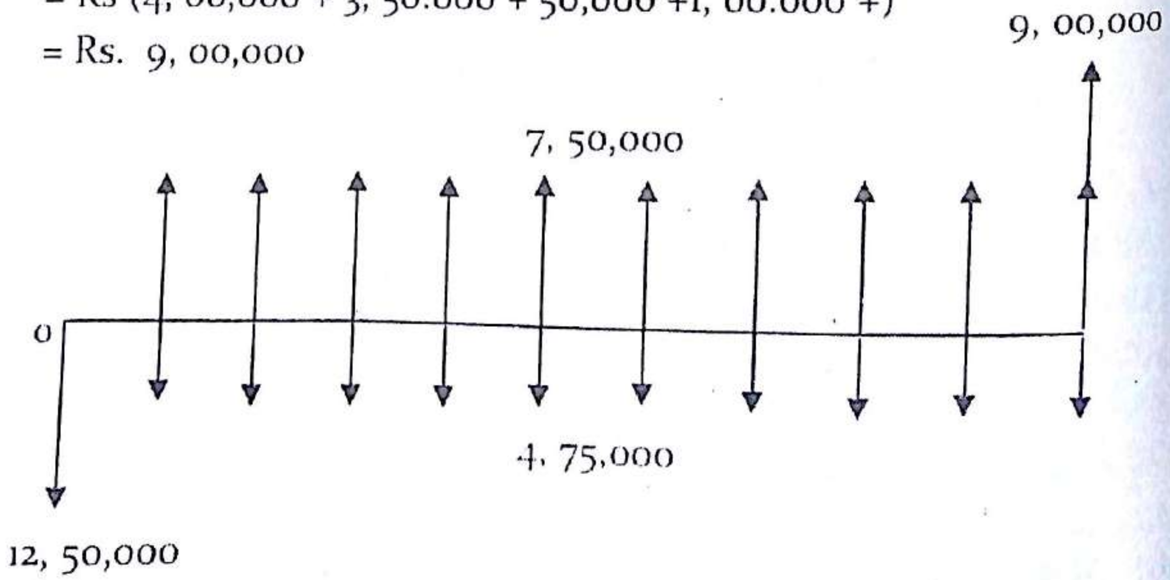
Solution

Total Investment = Rs. (3, 00,000 + 6, 00,000 + 2, 50,000 + 1, 00,000) = Rs. 12, 50.000.

Total salvage value of after 10 years:

= Rs (4, 00,000 + 3, 50.000 + 50,000 +1, 00.000 +)

= Rs. 9, 00,000



PW (25%) = - 12, 50,000 +9, 00,000 (P/F, 25%, 10) + (7, 50,000-4, 75,000) (P/A, 25%, 10)

$$= -12, 50{,}000 + 9, 00{,}000\ (1+0.25)^{-10} + (7, 50000-4, 75000)\left[\frac{(1+0.25)^{10}-1}{(1+0.25)^{10}\times 0.25}\right]$$

= **Rs. – 1, 71,474.83**

Hence, PW (25%) < 0, the investment is not feasible or rejected //

Future worth Method (FW)

Net present worth measures the surplus in an investment project at time zero where as net future worth measures this surplus at time period other than zero.Net future worth analysis is particularly useful on an investment solution where we need to compute the equivalent worth of a project at the end of investment period rather than its beginning.

$$FW(i\%) = A_0 (1+i)^N + A_1 (1+i)^{N-1} + A_2 (1+i)^{N-2} + \text{--------} + A^N$$

$$= \sum_{n=0}^{N} A_n (1+i)^{N-n}$$

$$= \sum_{n=0}^{N} A_n (F/P, i\%, + N-n)$$

Decision rule

NFW with positive value is undertaken same as NPW.

If FW (i) > o'	*accept the investment*
If FW (i) = o'	*remain indifferent*
If FW (i) < o'	*reject the investment*

Example: 4.4

Consider the example 4.1 and calculate the economic feasibility of the machine using FW formulation.

Solution

Using the future worth formulation

FW (10%) = -100,000 (F/P, 10%, 12) + 15,000 (F/A, 10%, 12) +25,000

= -100,000 *3.1384 + 15,000 *21.3843 +25,000

= -3, 13,840 + 3, 20,764.5 + 25,000

= **Rs 31,924.5**

Since FW is positive; it is worthwhile to install the machine.

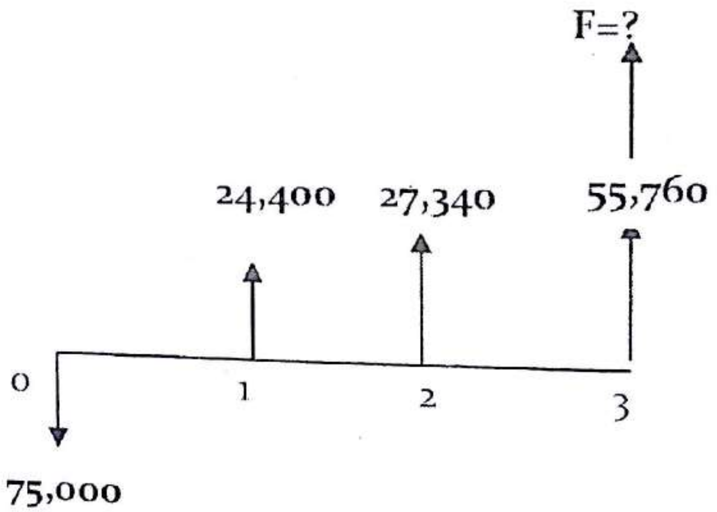
Example: 4.5

Consider the following cash flow for a project.

End of year	Net cash flow
0	- Rs 75,000
1	Rs 24,400
2	Rs 27,340
3	Rs 55, 760

MARR = 15%. Is the Investment accepted? Use FW formulation

Solution



The FW of the project at 15%

FW (15%) = - 75,000 (F/P, 15%, 3) + 24,400 (F/P, 15%, 2) + 27,340 (F/P, 15%, 1) + 55,760.

= **Rs 5,404**

Here, FW (15%) >0, the investment is economically accepted or justified.

Annual worth Method (AW)

Annual worth method provides the basis for measuring investment worth by determining equal payments on an annual basis. The AW of a project is its annual equivalent receipts (R) minus annual equivalent expenses (E) minus annual equivalent capital Recovery (CR). R, E, and CR are calculated at MARR

$$AW(i) = R - E - CR$$

Knowing that any lump sum cash amount can be converted into a series of equal annual payments we may first find the present worth of the original series and then multiply this amount by the capital recovery factor:

$$AW(i) = PW(i)\,(A/P, i, N)$$

Decision rule

NAW with positive value is undertaken same as NPW and NFW.

If AW (i) > o'	***accept the investment***
If AW (i) = o'	***remain indifferent***
If AW (i) < o'	***reject the investment***

Capital Recovery

When only cost are involved, the AW method is sometimes called the annual equivalent cost method. In this case, two types of costs are involved i.e. operating and capital cost. Operating cost (labors and raw materials etc) is recurred over the life of project and they are estimated on annual basis where as capital costs (purchasing assets or establishing company etc) tend to be one time cost. So for the purpose of annual equivalent cost analysis this onetime cost (capital cost) must be translated into its annual equivalent over the life of the project. This annual equivalent of capital cost is given a special name: Capital Recovery cost designated as CR (i). It covers depreciation and interest on invested capital. Two general monetary transactions are associated with the purchase and retirement of capital asset: Initial cost (I) and its Salvage value (S). The Capital Recovery cost can be calculated as follows:

$$CR = I\ (A/P,\ I\%,\ N) - S\ (A/F,\ i\%,\ N)$$

Example 4.6

Consider the example 4.1 and calculate the economic feasibility of the machine using AW formulation.

AW (10%) = - 100,000 (A/P, 10%, 12) + 15,000 + 25,000 (A/F, 10%, 12)

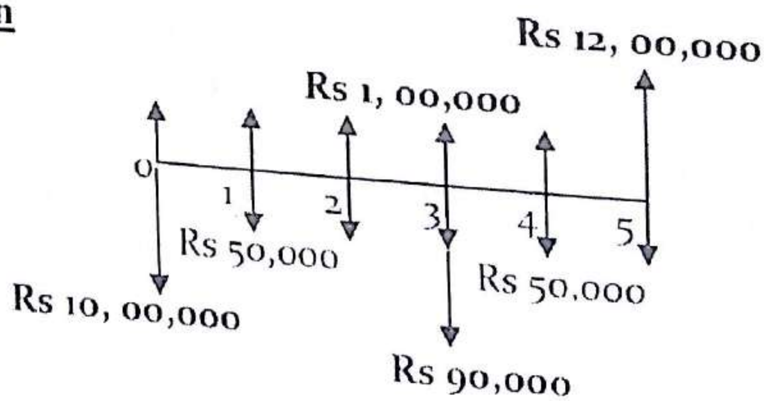
= -100,000 *0.1468 + 15,000 + 25,000 * 0.0468

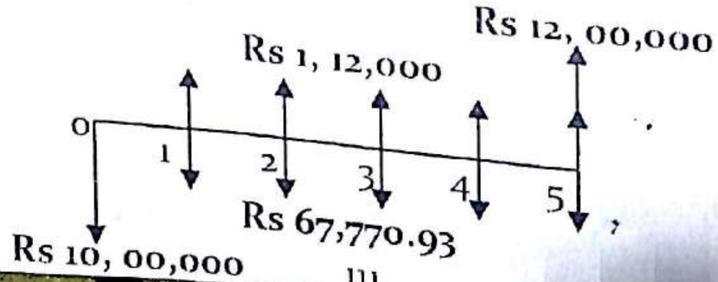
= Rs, 1,490. **Since AW is positive; it is worthwhile to install the machine**

Example 4.7

You purchased a building 5 years ago for Rs 10, 00,000. Annual Maintenance cost is Rs 50,000per year. At the end of 3 years Rs 90,000 was spent on roof repairs. At the end of 5 years you sell a building for Rs 12, 00,000. During the period of ownership, you rented the building for Rs. 1, 00,000 per year paid at the beginning of each year. Use AW method to evaluate the investment if MARR = 12%

Solution





Annual Revenue (R) = 10, 0000 $(1.12)^1$ = Rs112, 000
Annual Expenses (E) = 50,000 + {90,000 (P/F, 12%, 3)} (A/P, 12%, 5)

$= 50{,}000 + \{90{,}000\ (1+0.12)^{-3}\} \times [\frac{(1.12)^5 \times 0.12}{(1.12)^5 - 1}]$ = Rs 67,770.93

CR = I (A/P, I%, N) – S (A/F, i%, N)

$CR = 10,\ 00{,}000\ [\frac{0.12 \times (1.12)^5}{(1.12)^5 - 1}] - 12,\ 00{,}000\ [\frac{0.12}{(1.12)^5 - 1}]$

= Rs 88,533.25

AW (12%) = R- E - CR
AW (12%) = 1, 12,000 – 67,770.93 – 88,533.25
= **Rs – 44,304.18**

Here AW is – ve, the investment is not economically justified

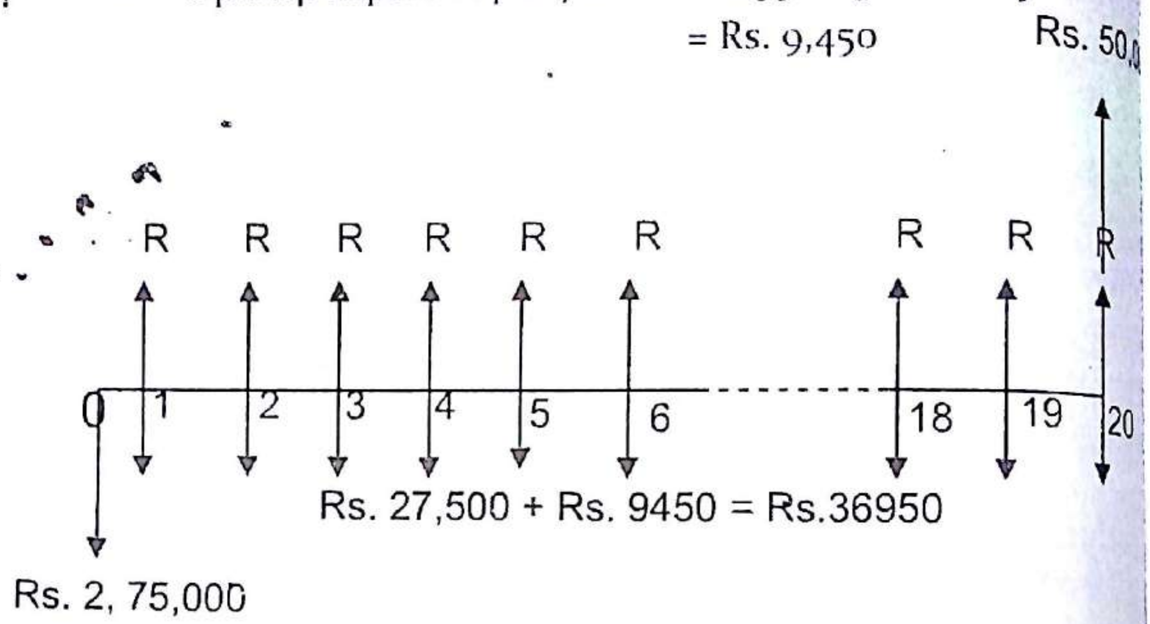
Example 4.8

An investment company is considering building a 25-unit apartment complex in a growing town. Because of the long term growth potential of the town, it is felt that the company could average 90% of the full occupancy for the complex each year. If the following items are reasonably accurate estimates, what is the minimum monthly rent that should be charged if a 12% MARR is desired? Use AW method.

- Land investment cost = Rs.50, 000
- Building investment cost = Rs. 2, 25,000
- Project life = 20 years
- Property tax and insurance per year = 10%of total investment
- Upkeep expenses per unit per month = Rs.35
- Salvage value = Rs. 50,000
- Rent per unit per month =?

Solution

- Total initial investment cost
  = Rs 50,000 + Rs 2, 25,000 = Rs 2, 75,000
- Taxes and insurance per year = 10% * Rs. 2,75,000
  = Rs. 27,500
- Upkeep expenses per year = Rs.35 * 25 * 12 * 0.9
  = Rs. 9,450



We Know

AW (i) = R – E – CR

Where, R = Required Revenue Per year

E = Annual Expenses

CR = Capital Recovery Cost

AW (12%) = R – 36,950 – CR

CR (12%) = 2, 75,000 (A/P, 12%, 20) – 50,000 (A/F, 12%, 20)

$$= 2,75,000\left[\frac{0.12\times(1.12)^{20}}{(1.12)^{20}-1}\right] - 50,000\left[\frac{0.12}{(1.12)^{20}-1}\right]$$

= Rs. 36,122.72

AW (12%) = R – 36,950 – 36,122.72

For no loss and no gain condition i.e. to accept the project marginally AW = 0.

AW (12%) = 0

R = 73072.72

The minimum monthly rent = 73,072.72 / 25*0.9*12

= **Rs. 2, 70.639 (Ans)**

4.3 Minimum Attractive Rate of Return

The minimum attractive rate of return (MARR) is the interest rate at which a firm can always earn or borrow money. MARR is the interest rate used in the time value of money calculation. It is generally dictated by management considering the following points:

- The amount of money available for investment - Source and cost of these funds (equity, borrowed funds etc.)
- The number of good project available for investment
- The amount of perceived risk associated with the investment
- The type of organization involved (government, public, private)

MARR is determined from the opportunity cost viewpoint, which results from the capital rationing phenomenon.

Capital rationing refers to the situation where the funds available for capital investment are not sufficient to cover potentially acceptable projects.

Opportunity cost: the best-rejected project or the worst accepted is the best opportunity foregone and its value is called the opportunity cost.

Determination of MARR

Consider the firm has

Available amount = Rs 60, 00,000

Seven projects available with total amount = Rs 75,00,000

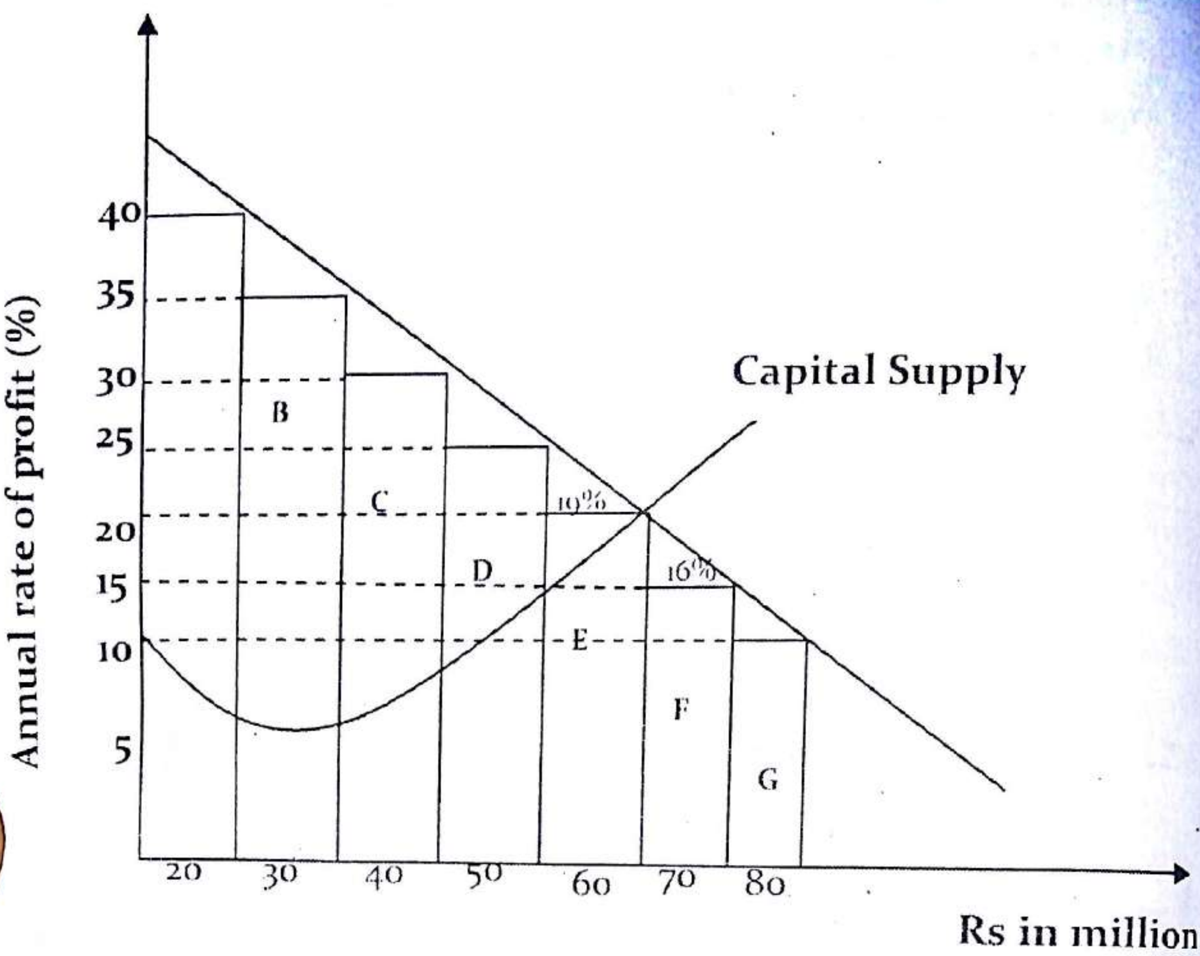


Fig 4.1: Determination of MARR

Last funded project = Project E

Prospective rate of profit = 19%

Best rejected project = F (by not being able to invest in project F, the firm would presumably be forfeiting the chance to realize on 16% annual return)

MARR = 16% per year (as the amount of investment capital and opportunities available change over time, the firm's MARR will also change)

Example

Suppose that you invested that amount (Rs.1, 650) in a savings account at 6% per year. Then, you could have only Rs.10, 648 on January, 2002. What is the meaning of this 6% interest here?

This is your opportunity cost if putting money in saving account was the best you can do at that time!
So, in 1970, as long as you earn more than 6% interest in another investment, you will take that investment.
Therefore, that 6% is viewed as a minimum attractive rate of return (or required rate of return).

4.4 Rate of Return Analysis

Definition 1:
Rate of return (ROR) is defined as the interest rate earned on the *unpaid balance* of an amortized (installment) loan
Suppose that bank lends Rs. 10,000, and it is repaid Rs. 4,021 at the end of each year for 3 years.
As studied in chapter 3 we have
10000= 4,021(P/A, i%, 3),
Solving for i we get i=10%.

The bank calculates the loan balances over the life of loan as follows:

Year	Unpaid balance at beginning year (Rs)	Return on Unpaid Balance	Payment received	Unpaid balance at end of year(Rs)
0	-10,000	0	0	-10,000
1	-10,000	-1,000	4,021	-6,979
2	-6,979	-698	4021	-3,656
3	-3,656	-366	4021	0

When the last payment is made the outstanding principal is eventually reduced to zero. If we calculate the NPW of the loan transaction at its rate of return (10%), we have,
PW (10%) = -10,000 +4021 (P/A, 10%, 3) = 0, this indicates that the bank can break even at a 10% rate of interest. The rate of

return becomes the rate of interest that equates the present value of future cash repayments to the amount of the loan.

Definition 2:

Rate of return (ROR) is the break-even interest rate, i^*, which equates the present worth of a project's cash outflows to the present worth of its cash inflows.

Mathematical Relation:

Using PW formulation

$PW(i^*) = 0$

$PW_{inflow} - PW_{outflow} = 0$

$$PW(i^*) = \sum_{k=0}^{N} R_k (P/F, i^*\%, k) - \sum_{k=0}^{N} E_k (P/F, i^*\%, k) = 0$$

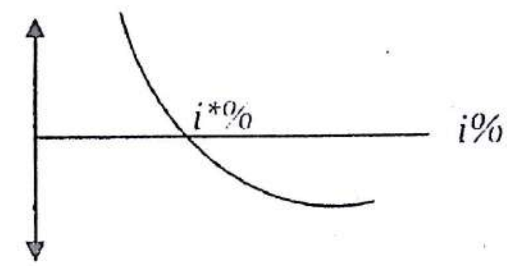


Fig 4.2 ROR

Using FW formulation

$$FW(i^*) = \sum_{k=0}^{N} R_k (F/P, i^*\%, N-k) - \sum_{k=0}^{N} E_k (F/P, i^*\%, N-k) = 0$$

Using AW formulation

$$AW(i^*) = \sum_{k=0}^{N} R_k (F/P, i^*\%, N-k)(A/F, i\%, N) - \sum_{k=0}^{N} E_k (F/P, i\%, N-k)(A/F, i\%, N) = 0$$

Where, R_k = Revenues in any time period k.

E_k = Expenses in any time period k

4.5 Internal Rate of Return (IRR)

A project's return is referred as *internal rate of return (IRR)* promised by an investment project over its useful life.

- *Internal rate of Return (IRR)* is the interest rate charged in the un-recovered project balance of the investment such that when the project terminates, the un-recovered project balance would be zero".
- In more familiar terms, the IRR of an investment is the interest rate at which the costs of the investment lead to the benefits of the investment. This means that all gains from the investment are inherent to the time value of money and that the investment has a zero net present value at this interest rate.
- IRR is the term for rate of return that stresses the interest earned on the portion of project that is internally invested.
- The unrecovered balance diagram is illustrated as below:

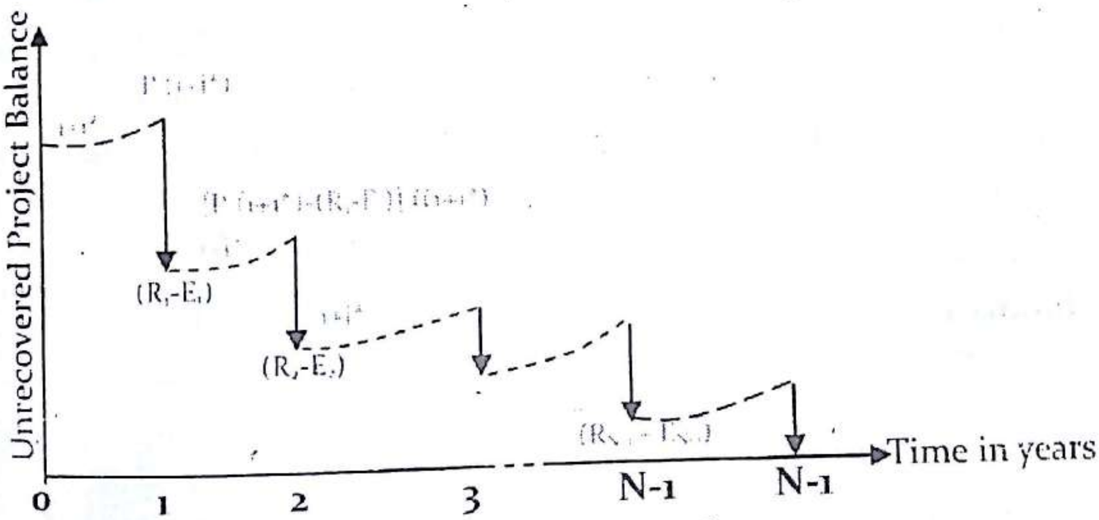


Fig 4.3: Investment Balance diagram

Here R_N and E_N are the Revenues and Expenses at the time period N

Example 4.13

Let us consider a company invests Rs 10, 000 in a computer and results in equivalent annual labor savings of Rs 4,021 over 3 years. The company is said to earn a return of 10% on its investment of Rs10, 000.

Project Balance Calculation

Year	Unpaid balance at beginning year (Rs)	Return on Unpaid Balance	Payment received	Unpaid balance at end of year (Rs)
0	-10000	0	0	-10,000
1	-10,000	-1,000	4,021	-6,979
2	-6,979	-698	4,021	-3,656
3	-3,656	-366	4,021	0

In the project balance calculation,

- 10% is earned (charged) on Rs.10, 000 during year 1.
- 10% is earned (charged) on Rs. 6,979 during year 2.
- 10% is earned (charged) on Rs. 3,656 during year 3

The firm earns a 10% rate of return on funds that remain internally invested in the project. Since the return is internal to the project, we call it internal rate of return (IRR). The computer project under consideration brings in enough cash to pay itself in 3 years and also to provide the firm with a return of 10% on its invested capital.

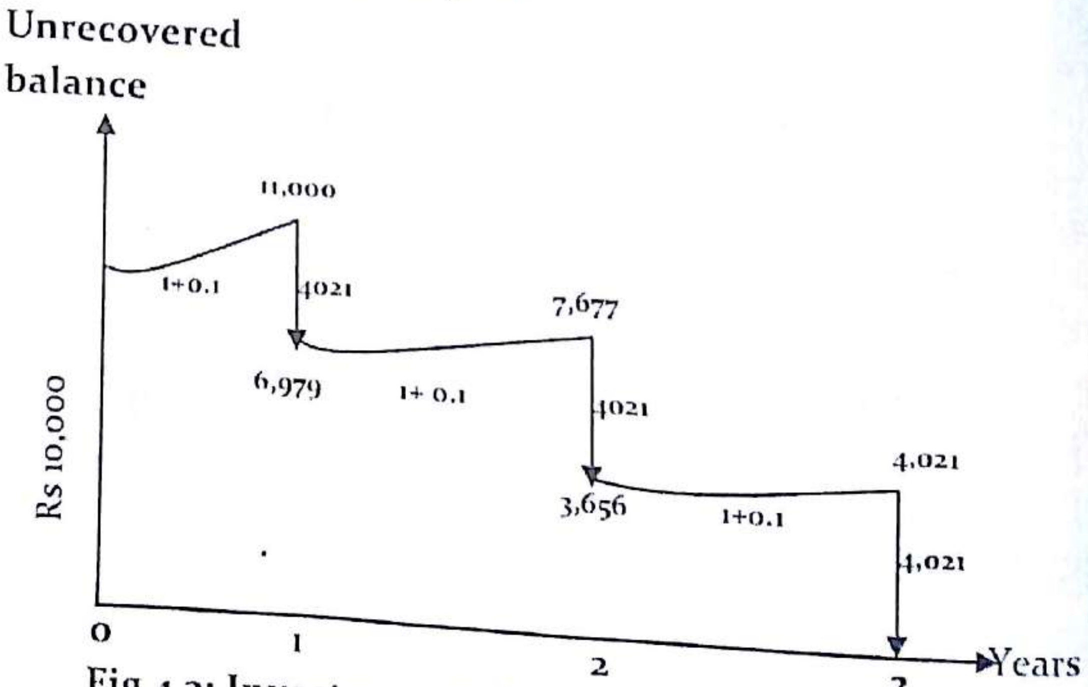


Fig 4.3: Investment Balance diagram of Example 4.7

Accept /Reject Decision Rule

If *IRR > MARR,* *accept the project*

$IRR = MARR$, *remain indifferent*

$IRR < MARR$, *reject the project*

Method of Finding IRR

Before dealing with the method for finding the IRR, first we need to identify the type of investment in a project. Once we have identified the types of investment there are several ways to find out the IRR. Mainly we can classify the investment in two types, they are; **Simple Investment and Non-simple Investment.** A *simple investment* is one in which initial cash flow is negative. Only one sign changes occur in the net cash flow series. A *non simple investment* is one in which initial cash flow is negative and more than one sign change occurs in the cash flow series.

Investment Type	Cash flow sign at period					
	0	1	2	3	4	5
Simple	-	+	+	+	+	+
Simple	-	-	+	+	0	+
Non Simple	-	+	-	+	+	-
Non simple	-	+	+	-	0	+

Example 4.14

Consider the three cash flow series shown below and classify them to simple and non simple investments

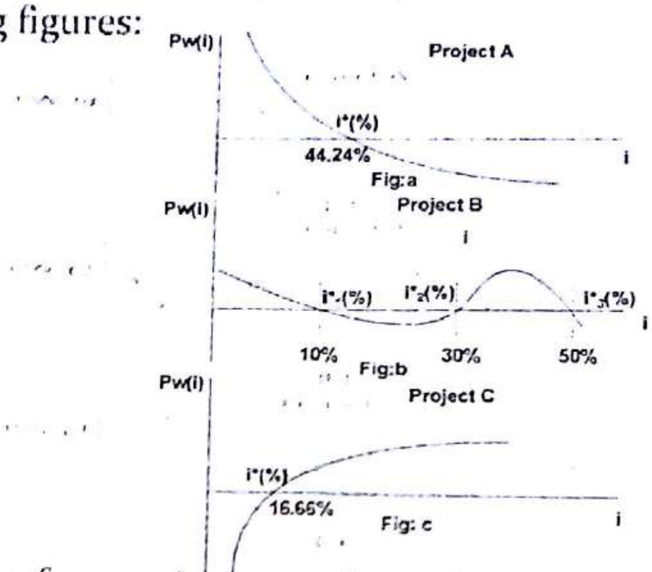
Period	Project A	Project B	Project C
0	- Rs. 1,000	- Rs. 1,000	Rs. 1,000
1	- Rs. 5,00	+ Rs. 3,900	-Rs. 450
2	+ Rs. 8,00	- Rs. 5,030	- Rs. 450
3	+ Rs. 1,500	+ Rs.2,145	- Rs. 450
4	+ Rs. 2,000		

As we can see in the table,

Project A is the Simple investment

Project B is the non –Simple investment
Project C is the simple borrowing

If we calculate the rate of return for each project, we obtain the following figures:



The IRR of a project can be calculated by the following methods.

1. **Direct Solution Method**
2. **Trial and error method**
3. **Computer Solution method**

1. Direct Solution Method

For the very special case of a project with only a two-flow transaction (an investment followed by a single future payment) or service life of 2 years of return, we can apply direct mathematical solution for determining the rate of return.

Example 4.15

Period (n)	Project 1	Project 2
0	- Rs1,000	-Rs2,000
1	Rs0	Rs1,300
2	Rs0	Rs 1,500
3	Rs0	
4	Rs1500	

...e rate of return for each proj...

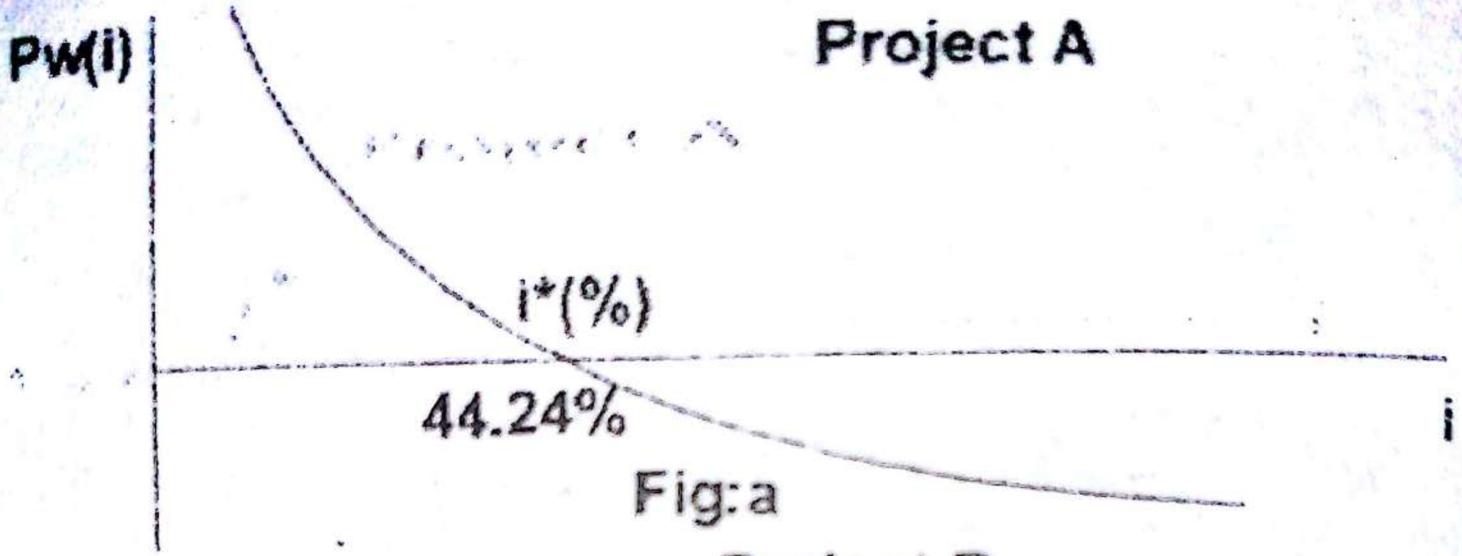


Fig:a

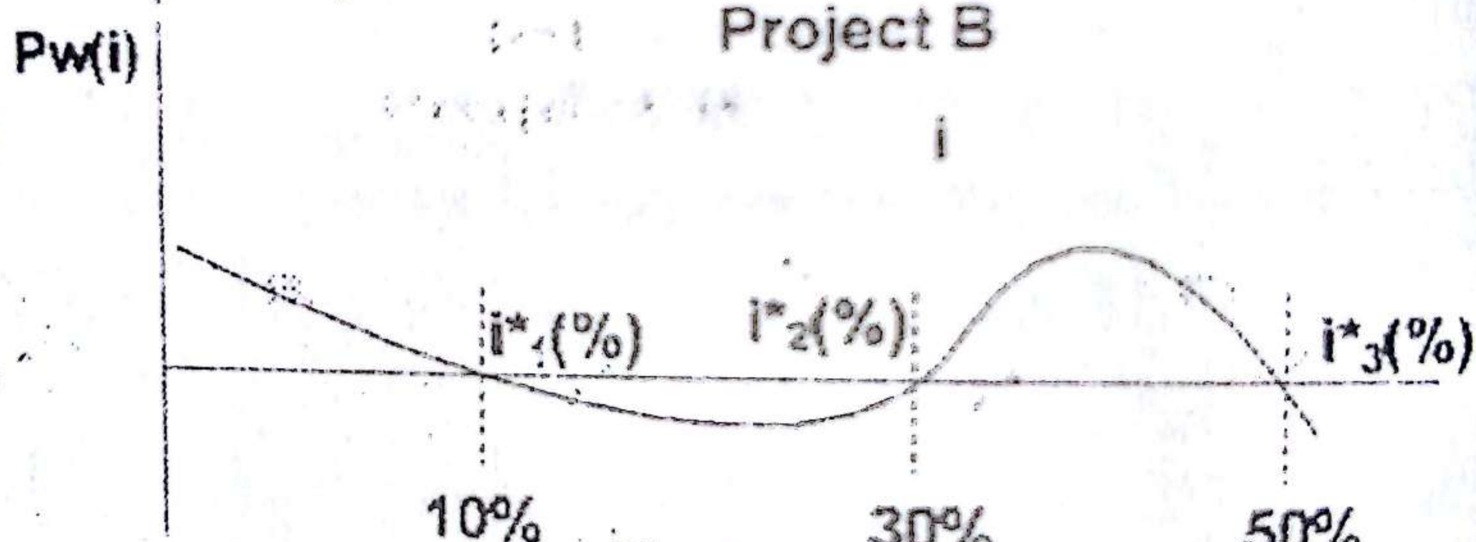


Fig:b

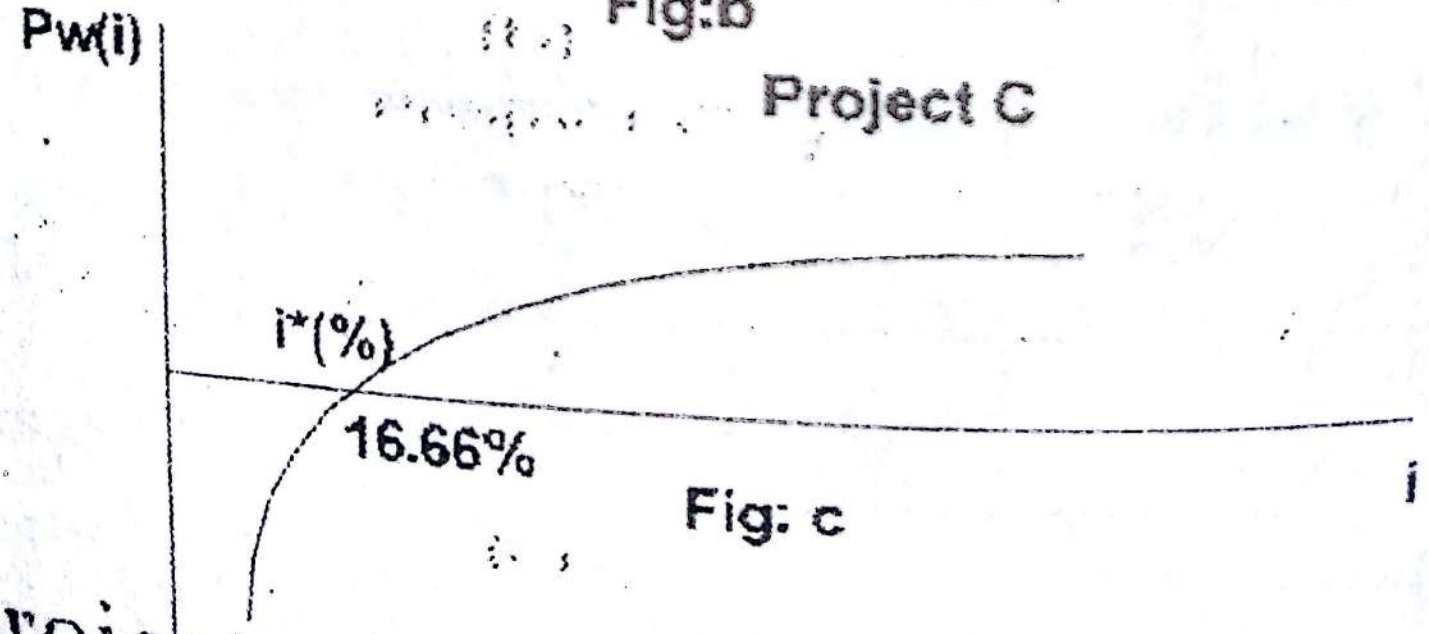


Fig: c

Project can be calculated b...

Consider two investment projects with the cash flow transactions as above. Compute the rate of return for each project.

Project A

$$Rs1{,}000 = Rs1{,}500(P/F, i, 4)$$

$$Rs1{,}000 = Rs1{,}500(1+i)^{-4}$$

$$0.6667 = (1+i)^{-4}$$

$$\frac{\ln 0.6667}{-4} = \ln(1+i)$$

$$0.101365 = \ln(1+i)$$

$$e^{0.101365} = 1+i$$

$$i = e^{0.101365} - 1$$

$$= 10.67\%$$

Project B

$$PW(i) = -Rs2{,}000 + \frac{Rs1{,}300}{(1+i)} + \frac{Rs1{,}500}{(1+i)^2} = 0$$

Let $x = \frac{1}{1+i}$, then

$$PW(i) = -2{,}000 + 1{,}300x + 1{,}500x^2$$

Solve for x:

$x = 0.8$ or -1.667

Solving for i yields

$$0.8 = \frac{1}{1+i} \rightarrow i = 25\%, \quad -1.667 = \frac{1}{1+i} \rightarrow i = -160\%$$

Since $-100\% < i < \infty$, the project's $i^* = 25\%$.

1. Trial and error method

Step 1: Guess an interest rate, say i =15%

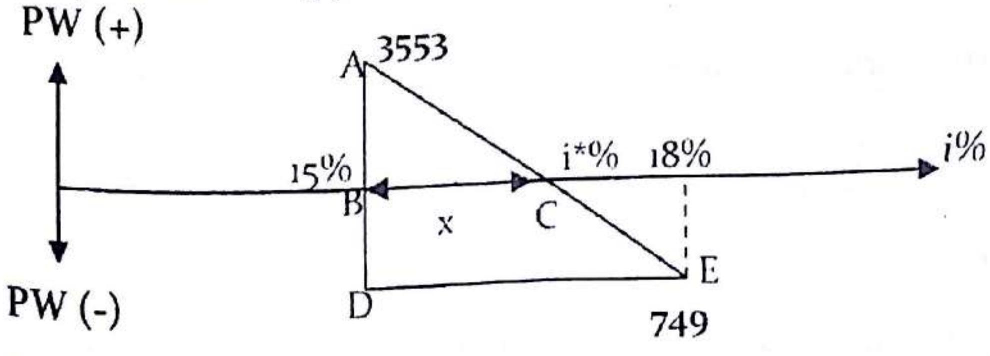
Step 2: Compute PW (i) at the guessed i value. (Consider the project C of example 4.8)

PW (15%) = Rs 3,553

Step 3: If PW (i) > 0, then increase i. If PW (i) < 0, then decrease i.

PW (18%) = - Rs 749

Step 4: If you bracket the solution, you use a linear interpolation to approximate the solution.



Here, i*% = 15% + BC

Using the equilateral triangle,

BC/DE = AB/AD

$x/(18-15)\% = 3553/(3553+749)$

$x = 3\% * [3553/(3553+749)] = 2.45\%$

$i^*\% = 15\% + 2.45\% = 17.45\%$

If i^* > MARR, select the project.

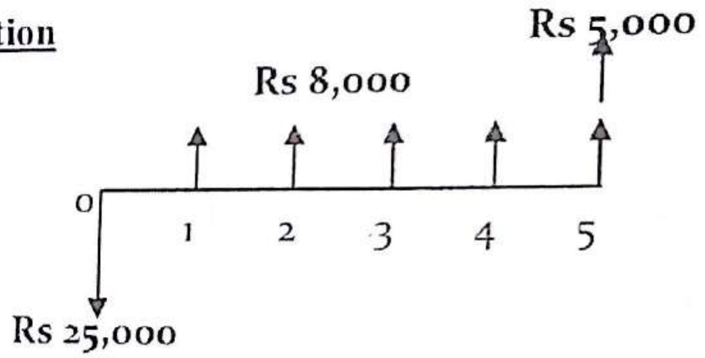
If i^* < MARR, reject the project

This rule does not work for a situation where an investment has multiple rates of return

Example 4.16

Consider the project with the initial investment = Rs 25,000, salvage value after 5 years = Rs 5,000, Net revenue = Rs.8, 000 / year. Is the investment good? MARR = 20%. Use IRR method.

Solution



Using PW formulation

PW (i^*) = 0

PW inflow – PW outflow = 0

5,000 (P/F, i^*15) + 8000 (P/A, i^*, 5) = 25,000

$$5{,}000(1/(1+i^*)^5)+8{,}000\frac{(1+i^*)^5-1}{i^*(1+i^*)^5}-25{,}000=0 \quad \text{.................. (1)}$$

By trial and error

At i^*= 20%, PW = 934.30

i^*= 25% PW = -1847.10

Using linear interpolation

$$\frac{25\%-20\%}{934.30-(-1847.10)} = \frac{i^*\%-20\%}{934.30-0}$$

i^* = 21.679%

Putting i^* = 21.679% in eq.1, PW (21.679%) = -58.20

IRR lies between 20% and 21.679%

Again using linear interpolation

$$\frac{21.679\% - 20\%}{934.30 - (-58.20)} = \frac{i^* - 20\%}{934.30 - 0}$$

$i^* = 21.58\%$

Putting $i^* = 21.58\%$ in eq.1

PW (21.58%) = -1.40

IRR lies between 20% and 21.58%

Again, interpolating

IRR, $i^* = 21.577\%$, where PW (21.577%) = 0

Here, IRR > MARR, investment is acceptable

Unrecovered Project Balance Calculation

Year	Unpaid balance at beginning year (Rs)	Return on Unpaid Balance (Rs)	Payment received	Unpaid balance at end of year (Rs)
0	-25,000	0	0	-25,000
1	-25,000	-5,394.25	8,000	-22,394.25
2	-22,394.25	-4,832	8,000	-19,226.25
3	-19,226.25	-4,148.45	8,000	-15,374.70
4	-15,374.70	-3317.40	8,000	-10,692
5	-10,692	-2307	13,000	0

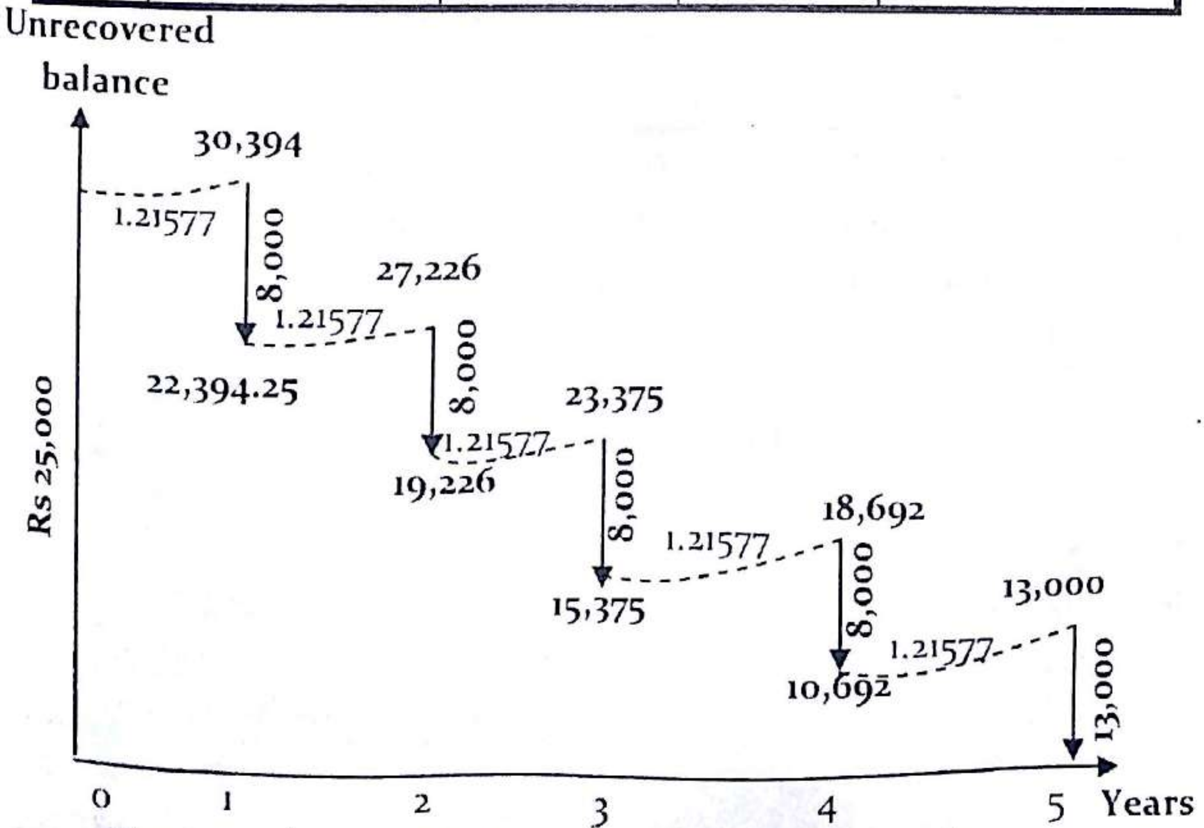


Fig: Investment Balance diagram of example 4.10

Example 4.17

Consider the investment project which has a project cash flow as follows. Is the project feasible if MARR = 10%. Use IRR method and draw investment balance diagram.

End of year	Net Cash flow
0	- Rs 45,000
1	-Rs 4250
2	Rs 9280
3	Rs 38600
4	Rs 61460
5	-Rs 20220

Solution

Using PW formulation

PW inflow – PW outflow = 0

Or, 4250 (P/F, i*, 1) + 92800 (P/F, i*, 2) + 38600 (P/F, i*, 3) +61460 (P/F, i*, 1) - 20220 (P/F, i*, 5) – 45,000 =0

By solving we get i*% = 21.40%

Unrecovered Project Balance Calculation

Year	Unpaid balance at beginning year (Rs)	Return on Unpaid Balance	Payment received (Rs)	Unpaid balance at end of year (Rs)
0	-45,000	0	0	-45,000
1	-45,000	-9,630	-4250	-58,880
2	-58,880	-12,600.32	9,280	-62,200.32
3	-62,200.32	-13310.86	38,600	-36,911.18
4	-36,911.18	-7899	61460	16649.8
5	16649.8	3563.06	-20,212≈20,200	0

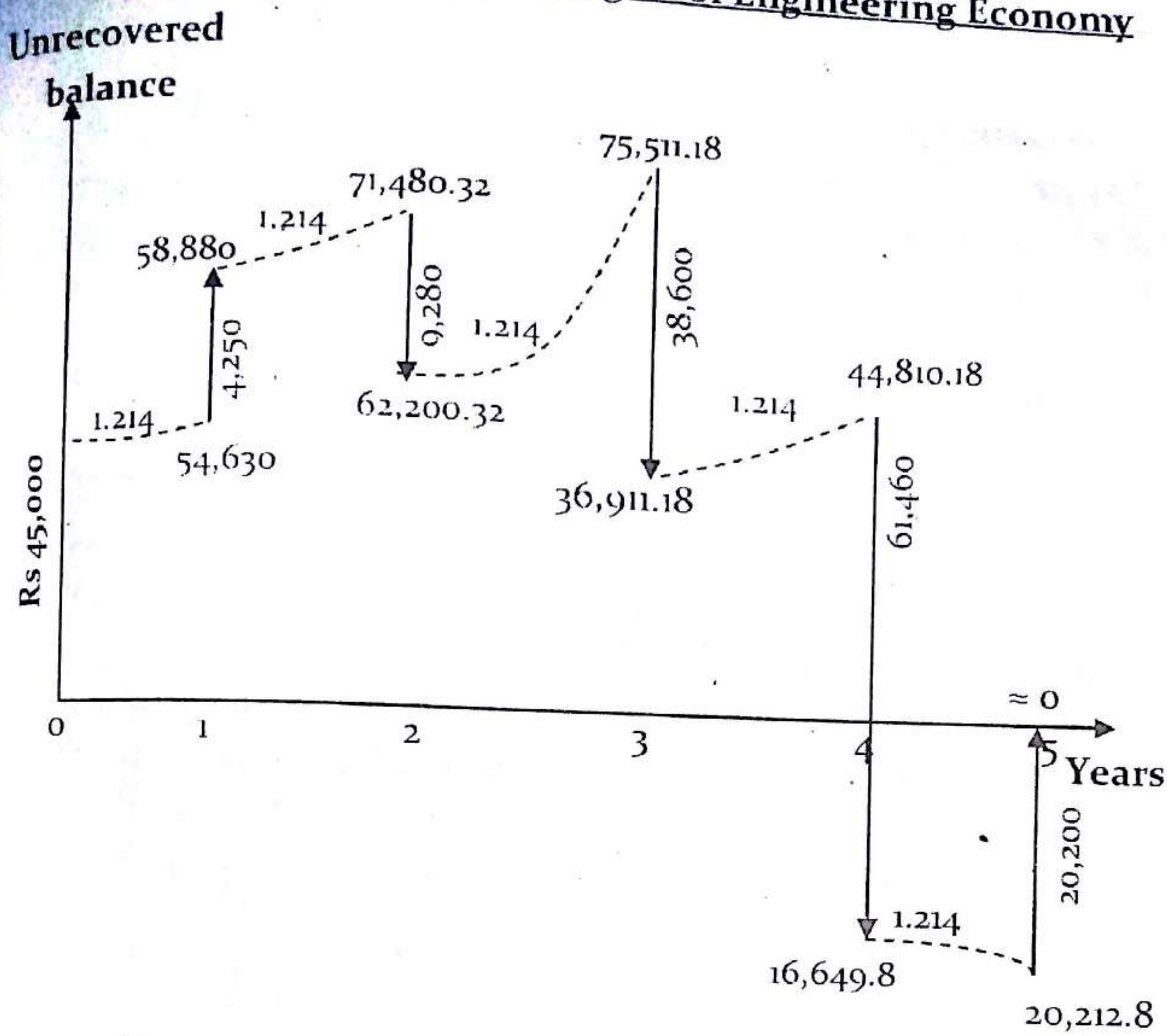


Fig: Investment balance diagram of example 4.11

2. **Computer Solution Method**

Consider the example 4.12,

PW (i* %) = 0

8,000 (P/A, i*%, N) + 5,000 (P/F, i*%, N) -25,000 = 0

$8{,}000\ ((1+i^*)^5 - 1)/\ ((1+i^*)^5 * i) + 5{,}000\ (1+i^*)^{-5} - 25{,}000 = 0$

$8\ ((1+i^*)^5 - 1)/\ ((1+i^*)^5 * i^*) + 5\ (1+i^*)^{-5} - 25 = 0$

Write the equation in calculator as

$8\ ((1+A)^5 - 1)/\ ((1+A)^5 * A) + 5\ (1+A)^{-5} - 25 = 0$

Press SHIFT → CALC

It may take some time to calculate. The value in the calculator gives the value of A and this is the IRR.

In Microsoft Excel 2007and 2010,

Click Formula

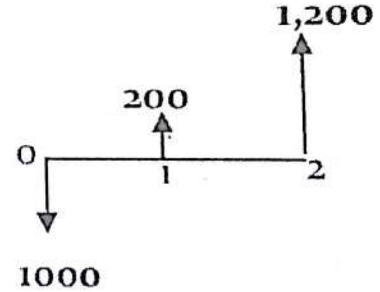
Click Auto Sum

Click More Function
Type IRR in Search for a Function, Click Ok
Insert the value of Cash Flow
Click Enter,
The result gives the value of IRR.

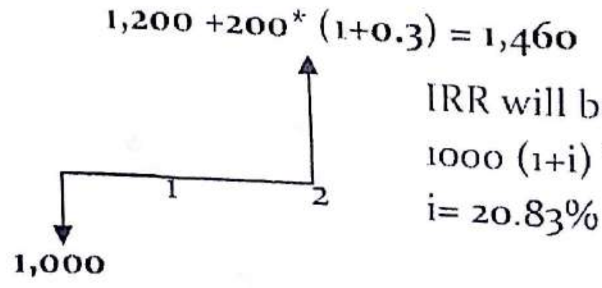
Drawbacks of IRR

Using IRR is common. Unfortunately, its misuse is also common. Although net present value is a more stable metric, the use of IRR is common, despite its shortcomings. Investors should know all the drawbacks of the IRR and when using it is inappropriate. The drawbacks of the IRR are:

1. It assumes all inter temporal cash flows are reinvested at its own rate, usually higher than is possible. This leads to the concept of External Rate of Return (ERR). Let us consider the following cash flow diagram



For the above cash flow pattern, we find IRR =20%, but at the end of 1st year, recovered fund is Rs 200 which may be reinvested in other than 20%. If it is invested in say 30%. The new cash flow diagram will be



IRR will be

$1000\,(1+i)^2 = 1{,}460$

$i = 20.83\%$

2. IRR method involves linear interpolation of non linear function and when solved manually by trial and error

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method, may not give accurate result and it is more time consuming.

3. There are situations in which its iterative calculation process fails to produce a solution.
4. When the algebraic sign of the cash flow changes in the middle of the series it is possible to obtain two "right" answers.

Let us consider the following cash flow

EOY	Net Cash Flow
0	-1,000
1	+2,300
2	-1,320

From the above cash flow pattern, we find IRR =10% and 20%, but both of them are incorrect. So we may abandon the IRR method for practical purpose and use the NPW criterion to make the decision.

5. When mutually exclusive projects are considered it can recommend the wrong investment and does not consider the scale of the investment.

Let us consider the two mutually exclusive projects.

EOY	Mutually Exclusive Alternatives	
	A	B
0	-Rs. 100	-Rs 500
1	+Rs. 150	+Rs 650
IRR	50%	30%
PW (10%)	36	90.90

Here both projects are acceptable at MARR =10%, but project B with higher PW is more worthful, whereas from IRR point of view, Project A seems better. Since IRR is a relative measure of investment worth, this inconsistency in ranking occurs. Under this circumstance, incremental analysis is necessary.

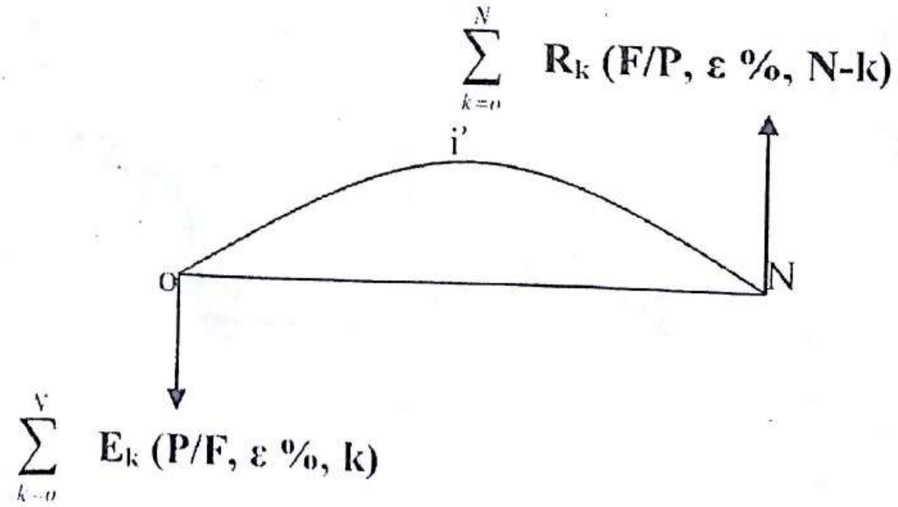
4.6 External Rate of Return (ERR)/ Modified IRR

The drawback of the IRR method (reinvestment assumption) may not be valid in the engineering economy. For example, if a firm's MARR is 20% per year and the IRR for a project is 42.4%, it may not be possible for the firm to reinvest net cash proceeds from the project at much more than 20%. This situation, coupled with the computational demands and possible multiple interest rates associated with the IRR method, has given rise to other rate of return methods that can remedy this weakness which is referred as *External rate of Return* or *Modified IRR*.

The external Rate of Return (i') is the unique rate of return for a project that assumes that net positive cash flows, which represent money not immediately needed by the project, are reinvested at the reinvestment rate ε %. The reinvestment rate depends upon the market rate available for investments.

Steps of ERR calculation

- All cash outflows are discounted to period zero (present) at ε % per compounding period
- All cash inflows are Compounded to period N at ε %
- ERR is the interest rate that equivalence between the two equation.



$$\sum_{k=0}^{N} E_k (P/F, \varepsilon\ \%, k)(F/P, i', N) = \sum_{k=0}^{N} R_k (F/P, \varepsilon\%, N-k)$$

Where,

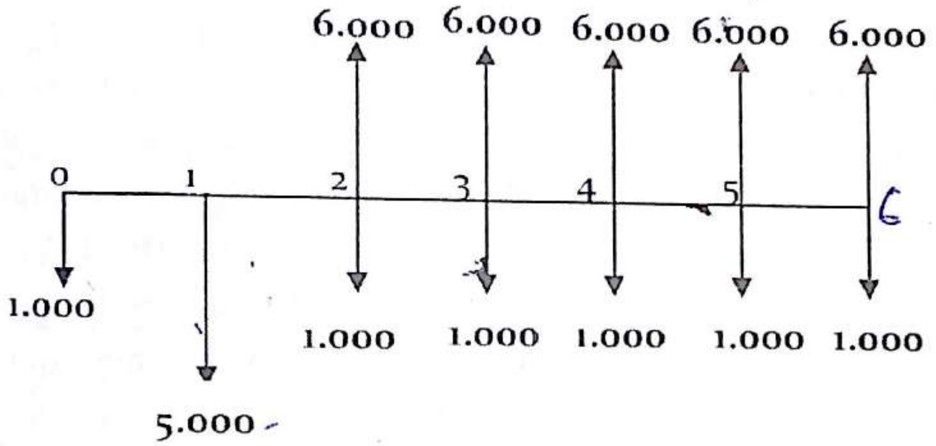
R_k = receipts in period k
E_k = expenditures in period k
N = project life or number of study period
ε % = external reinvestment rate per period.

Accept /Reject Decision Rule

If *ERR > MARR,* *accept the project*
ERR = MARR, *remain indifferent*
ERR < MARR, *reject the project*

Example 4.18

Consider the following cash flow of the project.



Calculate the ERR of the project if MARR = 20% and reinvestment rate ε % =15%. Is the project accepted?

Solution

1. Discounting all the cash outflows to the time zero at 15%.

1,000 + 5,000 (P/F, 15%, 1) = 1,000 + 5,000 $(1+0.15)^{-1}$ = 5,347.82

2. Compounding all the cash inflows to the year 6 at 15%

   5,000 (F/A, 15%, 5) = 5,000 $\{(1.15)^5 -1 / 0.15\}$ = 33,711.90

3. Establishing the equivalence between the two equation

   5,347.82 (F/P, i', 6) = 33,711.90

   5,347.82 $(1+i')^6$ = 33,711.90

   $(1+i')^6$ = 6.303

   i' = 1.359 -1 = **35.91%** is the ERR of the project.

Here ERR (35.91%) > MARR (20%), the project is accepted.

Advantages of ERR over IRR

- It does not need trial and error approach for determination of i*%
- There is no possibility of multiple rate of return.

4.7 Benefit Cost Analysis (B/C ratio)

Benefit-Cost-Analysis is a decision making tool used to systematically develop useful information about the desirable and undesirable effects of public project. Benefit Cost Analysis estimates and totals up the equivalent money value of the benefits and costs to the community of projects to establish whether they are worthwhile. These projects may be dams, irrigation, water supply highways etc. or can be training programs and health care systems. In other words the benefit/cost ratio is defined as the ratio of the equivalent worth of benefits to the equivalent worth of costs. The equivalent worth measure applied can be present worth (PW), future worth (FW), annual worth (AW). It is also called *"savings-investment ratio"*. To perform an economic analysis of public alternatives, the cost (initial and annual), the benefits and the dis-benefits, if considered, must be estimated as accurately as possible in monetary units. It is difficult to estimate and agree upon the economic impact of benefits and dis-benefits for a public sector alternative. Dis-benefit may not be known at the time of economic analysis.

Two types of B/C ratio

1. ***Conventional B/C ratio***
2. ***Modified B/C ratio***

Benefits – advantages to be experienced by the owners, the public. (Favorable outcomes)

Dis-benefits – expected undesirable or negative consequences to the public, if the alternative is implemented. (Unfavorable outcomes)

User's benefit = Benefits – Dis-benefits

Cost: estimated expenditures to the government entity for construction, operation and maintenance of the project, less any expected salvage value. Include capital investment and annual operating costs.

Sponsor's Cost = Capital cost + Operating and Maintenance costs - Revenues

Public Sector Project

Public sector project are owned, used and financed by the citizenry of any government level. Public sector projects have a primary purpose to provide services to the citizenry for the public good at no profit. Areas such as health, safety, economic welfare, and utilities comprise a majority of alternatives that require engineering economic analysis. Usually public sector investment decisions involve a great deal of expenditure, and their benefits are expected to occur over an extended period of time. One of the important issues that the toll authority must address is how it can determine whether its decisions, which affect the use of public funds, are, in fact, in the best public interest. In identifying the benefits of a project of this nature, we need to consider both primary benefits –the ones directly attributable to the project- and the secondary benefits – the ones indirectly attributable to the project.

Accept /Reject Decision Rule

For the project to be feasible

Benefit (B) > Cost (C)

Or B/C > 1

If *B/C > 1,* *accept the project*

B/C = 1, *remain indifferent*

B/C < 1, *reject the project*

1. **PW method**

Conventional B/C ratio = $\dfrac{\text{PW (benefits of proposed project)}}{\text{PW (total cost of proposed project)}}$

$$= \frac{PW(B)}{(I) - PW(S) + PW(O\&M)}$$

$$\text{Modified B/C ratio} = \frac{PW(B) - PW(O\&M)}{(I) - PW(S)}$$

Where, PW (B) = Present worth of benefits of project.

PW (O&M) = Present worth of Operation and maintenance of project.

PW (S) = Salvage value of investment.

I = Initial investment of the project.

2. **FW method**

$$\text{Conventional B/C ratio} = \frac{FW(B)}{FW(I) - (S) + FW(O\&M)}$$

$$\text{Modified B/C ratio} = \frac{FW(B) - FW(O\&M)}{FW(I) - (S)}$$

Where,

FW (B) = Future worth of benefits of project.

FW (O&M) = Future worth of Operation and maintenance of project.

PW (S) = Salvage value of investment.

I = Initial investment of the project.

3. **AW method**

$$\text{Conventional B/C ratio} = \frac{AW(\text{benefits of proposed project})}{AW(\text{total cost of proposed project})}$$

$$= \frac{AW(B)}{CR + AW(O\&M)}$$

$$\text{Modified B/C ratio} = \frac{AW(B) - AW(O\&M)}{(CR)}$$

Where,

AW (B) = Annual worth of benefits of project.

CR = Capital Recovery.

AW (O&M) = Annual worth of operation and maintenance cost

I = Initial investment of the project.

Note: Whichever method we use, value remains same, similarly

$B/C_{modified} > B/C_{conventional}$, if $B/C > 1$

$B/C_{modified} < B/C_{conventional}$, if $B/C < 1$

$B/C_{modified} = B/C_{conventional}$, if $B/C = 1$

Example 4.19

Find both types of B/C ratio using PW, FW and AW method.

Initial investment	=	Rs 20,000
Revenue / year	=	Rs 10,000
Expenses/ year	=	Rs 4,400
Salvage value	=	Rs 4,000
Useful life	=	5 years
MARR	=	8%

Solution

Using PW method

PW (B) $= 10{,}000\ (P/A,\ 8\%,\ 5) = 10{,}000\ \left[\frac{(1.08)^5 - 1}{(1.08)^5 * 0.08}\right]$

$= \text{Rs } 39{,}927$

PW (S) $= 4{,}000\ (P/F,\ 8\%,\ 5)$

$= 4{,}000\ (1/\ (1.08)^5)$

$= \text{Rs } 2{,}722.33$

PW (O&M) $= 4{,}400\ (P/A,\ 8\%,\ 5)$

$= 4{,}400\left[\frac{(1.08)^5 - 1}{(1.08)^5 * 0.08}\right]$

$= \text{Rs } 17{,}567.90$

Conventional B/C ratio $= \frac{PW\ (B)}{(I) - PW\ (S) + PW\ (O\&M)}$

$= \frac{39{,}927}{20{,}000 - 2{,}722.33 + 17{,}567.90}$

$= \mathbf{1.146 > 1}$ **(justified)**

Modified B/C ratio $= \dfrac{PW(B) - PW(O\&M)}{(I) - PW(S)}$

$= \dfrac{39{,}927 - 17567.90}{20{,}000 - 2722.33}$

$=$ **1.294 > 1 (justified)**

Using AW method

Conventional B/C ratio $= \dfrac{AW(B)}{CR + AW(O\&M)}$

CR $= 20{,}000\,[A/P, 8\%, 5] - 4{,}000\,(A/F, 8\%, 5)$

$= 20{,}000 \left[\dfrac{(1.08)^5 * 0.08}{(1.08)^5 - 1}\right] - 4{,}000\,(0.08/\,(1.08)^5 - 1)$

$=$ Rs 4,327

Conventional B/C ratio = 10,000/4,327+ 4,400

= **1.146 > 1 (justified)**

Modified B/C ratio $= \dfrac{AW(B) - AW(O\&M)}{(CR)}$

= 10,000 – 4,400 /4,327

= **1.294 >1 (justified)**

4.8 Payback Period Method (PB)

Payback considers the initial investment costs and the resulting annual cash flow. The payback time (period) is the length of time needed before an investment makes enough to recoup the initial investment. The payback method screens the projects on the basis of how long it takes for net receipts to equal investment. But the payback method doesn't account for savings after the initial investment is paid back from the profits (cash flow) generated by the investment (project).

Simple payback period

Simple payback period doesn't considers the time value of money (i=0). Simple Payback period is calculated using the following equation if the annual savings are equal:

$$Simple\ Payback\ Period\ (in\ years) = \frac{Initial\ Investment}{Annual\ Savings\ (Cashinflow)}$$

Where:

Initial Investment = Initial investment for a project

Annual Savings (Cash Flow) = Annual savings derived from the investment

In other way, the simple payback period for a project having one time investment at time zero can be computed as follows:

$$\text{Simple payback period: } (\theta) = \sum_{k=1}^{\theta} (R_k - E_k) - I \geq 0$$

Advantages of simple payback period

- Easy to calculate
- It is interpreted in tangible terms (in years)
- It doesn't require any assumptions about the project in terms of timing, life time or interest rates.

Disadvantages of simple payback period

- It takes no account of any savings after the payback period.
- It takes no account of the residual value in the capital asset.
- It takes no account of the time value of money.

Example 4.20

Consider an example of evaluating the purchase of pollution prevention equipment for a cost of Rs.8, 000, but provide a net annual operational saving of Rs.3, 500. When the net annual savings is divided into the initial investment, the simple payback period is calculated as follows:

$$Payback\ Period\ (in\ years) = \frac{Initial\ Investment}{Annual\ Savings\ (Cash\ Flow)} = \frac{8000}{3500}$$

$\approx 2.3\ (years)$

Example 4.16

Calculate the simple payback period for the given cash flow of the project.

Period	Net Cash flow (Rs)
0	-25,000
1	+8,000
2	+8,000
3	+8,000
4	+8,000
5	+13,000

Solution

Period	Net Cash flow (Rs)	Cumulative Cash Flow (Rs)
0	-25,000	-25,000
1	+8,000	-17,000
2	+8,000	-9,000
3	+8,000	-1,000
4	+8,000	+7,000
5	+13,000	+20,000

Here the cumulative Cash flow turns to positive in year 4. Therefore Payback period lie between year 3 and 4. By interpolating, we get the payback period = 3+1000/8000 = 3.125 **years (Ans)**

Discounted payback period

The problem with the Simple payback period is that it ignores the time value of money. In order to correct this, we can use discounted cash flows in calculating the payback period. Discounted Payback Period is one of several methods to determine if an investment is a good one or not. In this method, you discount each of the future cash flows and then count the number of years necessary to recoup your investment.

In other way, the discounted payback period for a project having one time investment at time zero can be computed as follows:

Discounted payback period (θ')

$$\sum_{k=1}^{\theta'} (R_k - E_k)\ (P/F, i\%, k) - I \geq 0$$

Advantages of discounted payback period

- Considers the time value of money
- Considers the riskiness of the project's cash flow (through the cost of capital)

Disadvantages of discounted payback period

- No concrete decision criteria that indicate whether the investment increases the firm's value.
- Requires an estimate of the cost of capital in order to calculate the payback.
- Ignores cash flows beyond the discounted payback period

Example 4.21

Consider the example 4.12 and Calculate the discounted payback period at i=20%

Solution

Discounted Period	Net cash flow (Rs)	PW of net cash flow	Cumulative Cash Flow

(n)		(i=20%)	(Rs)
0	-25,000	-25,000	-25,000
1	+8,000	6,667	-18,333
2	+8,000	5,556	-12,777
3	+8,000	4,630	-8,147
4	+8,000	3,858	-4289
5	+13,000	5,223	+934

Here the cumulative Cash flow turns to positive in year 5. Therefore Payback period lie between year 4 and 5. By interpolating, we get the payback period = 4+ 4,289/5,223 = **4.82 years (Ans)**

Some solved examples

1. Find IRR of the following project with the initial investment of 350,000 and the cash flow as shown. Also draw the investment balance diagram.

End of year	Cash Outflow	Cash Inflow
1	15,000	50,000
2	15,000	75,000
3	15,000	1,00,000
4	15,000	1,25,000
5	15,000	1,50,000

Solution

Net cash flow for each year

End of year	Net cash flow	A	G
1	35,000	35,000	0
2	60,000	35,000	25,000
3	85,000	35,000	50,000

4	110,000	35,000	75,000
5	135,000	35,000	100,000

Here the cash flow is in gradient series

Using gradient to present worth series factor

PW (i^* %) = 0

$-3,50,000 + 35,000\,(P/A,\, i^*,\, 5) + 25000\,(P/G,\, i^*,\, 5) = 0$

$-3,50,000+35,000[((1+i^*)^5-1)]/[i^*(1+i^*)^5] +25,000 \{((1+i^*)^5 -5^*i^*-1/ i^*2\,(1+i^*)^5\} = 0$

Solving by hit and trial, we get **IRR (i^* %) = 5.634%**

Unrecovered project balance

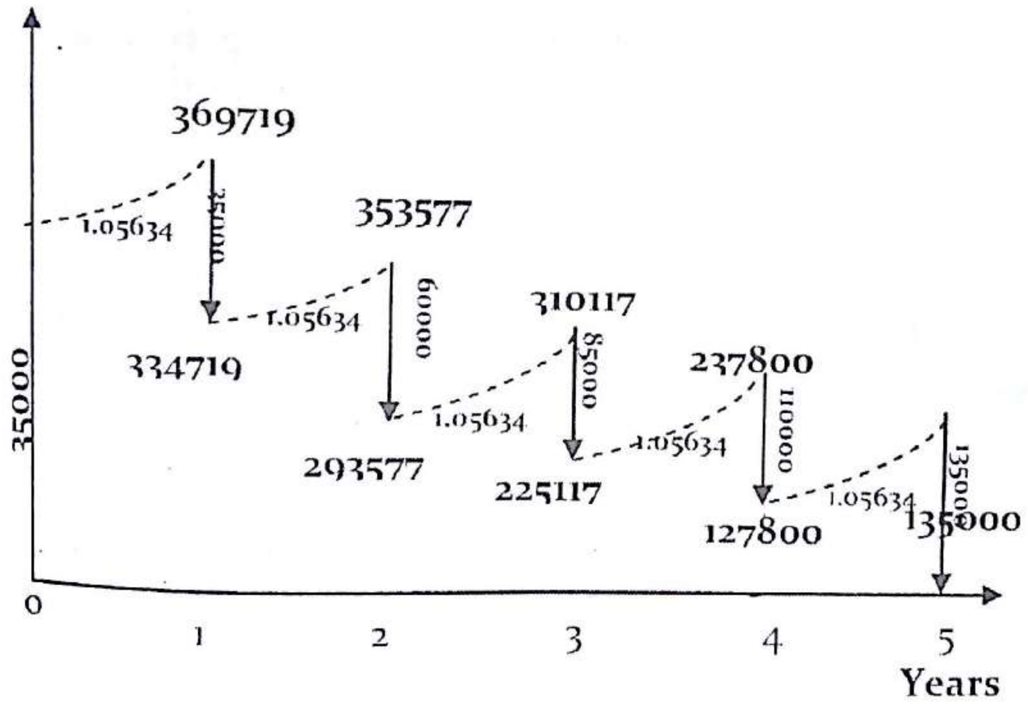


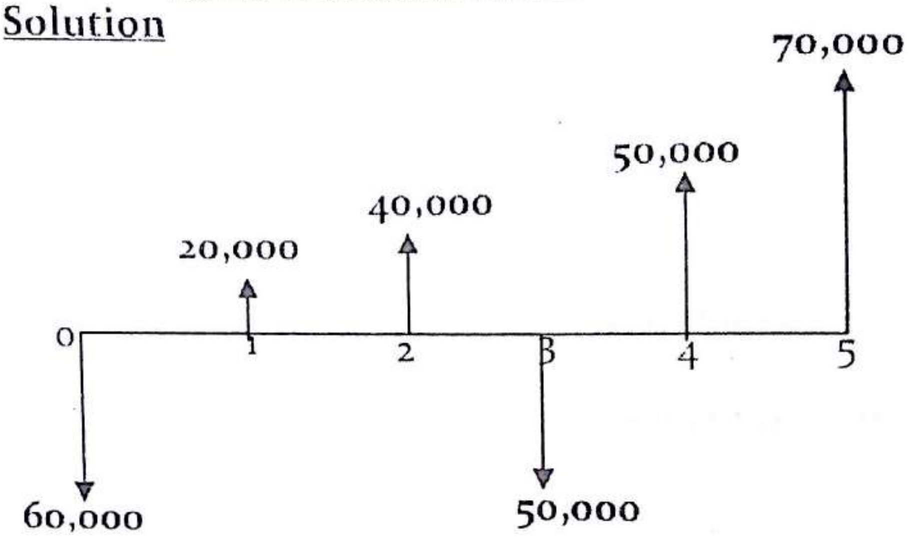
Fig: Investment balance Diagram

2. Find the ERR (external rate of return) when ε %=15%. **(TU, IOE, 2064)**

End of year	Annual Cash Flow
0	Rs - 60000

1	Rs +20000
2	Rs +40000
3	Rs - 50000
4	Rs +50000
5	Rs +70,000

Solution



Step 1: Discounting all the cash outflows to present time at ε =15%.

60,000 + 50,000 (P/F, 15%, 3) = Rs 92,875.8

Step 2: Compounding all the cash inflows to future time at ε =15%.

20,000 (F/P, 15%, 4) + 40,000 (F/P, 15%, 3) +50,000 (F/P, 15%, 1) + 70,000

= 34,980 + 60,835 + 57,500 + 70,000

= Rs 223,315

Step 3: Establishing the equivalence of the above two equation

92875.8 (F/P, i'%, 5) = 22, 3315

$(1+i')^5 = 2.40$

i' = 19.135%

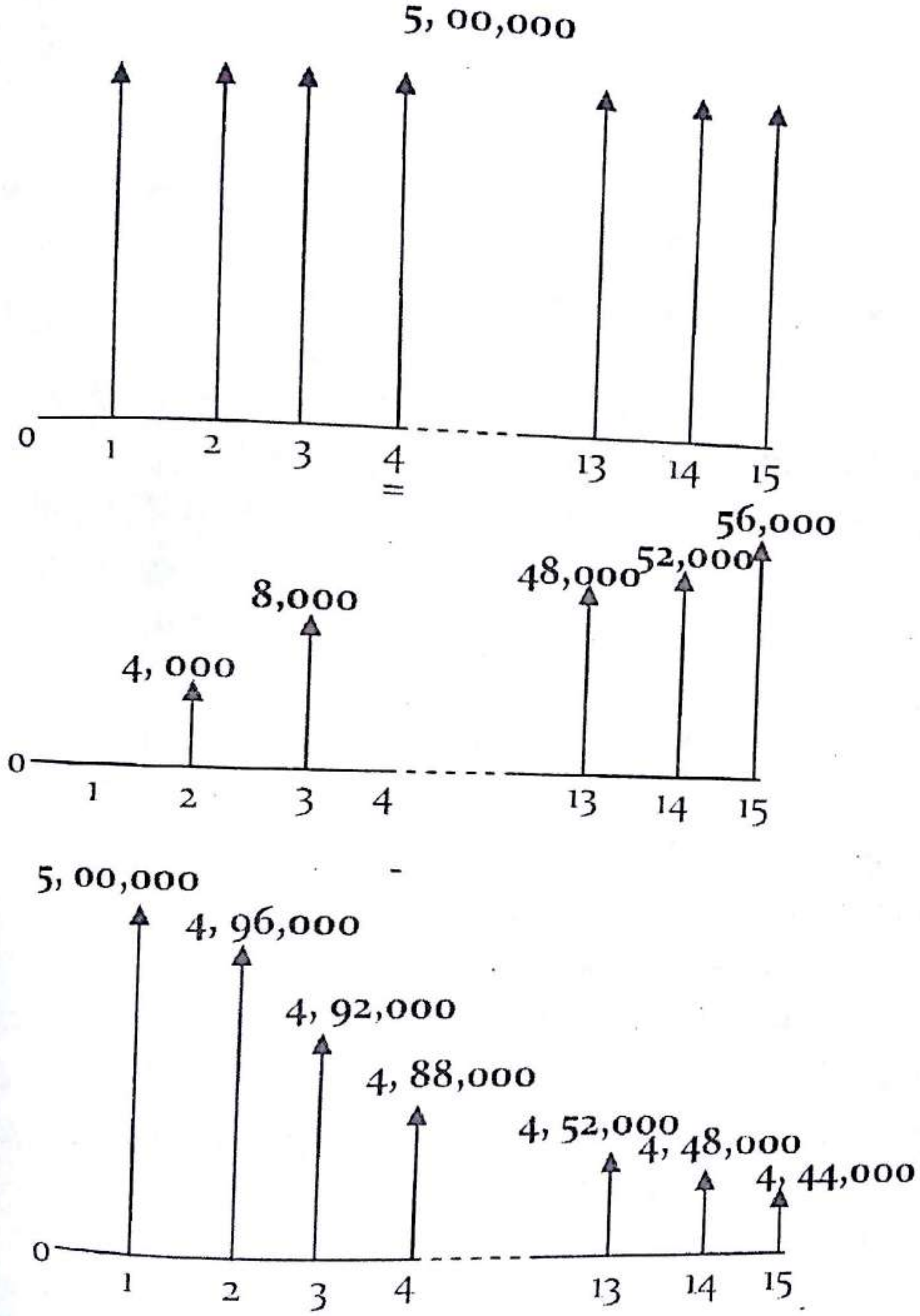
The external rate of return (ERR) is **19.315% (Ans)**

3. The annual income of the project starts from Rs 5, 00,000 at the end of first year and decreases at the rate of Rs 4000/yr for 15 years. What is the equivalent present worth when the MARR is 12%. **(TU,IOE, 2064)**

Solution:

Here, the cash flow is in decreasing gradient series.

N=15, G= 4000, i=12%,



Applying gradient to present worth series factor we get

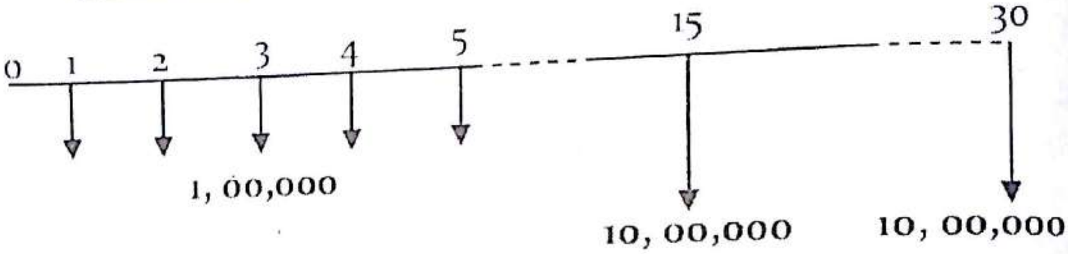
PW (12%) = 5, 00,000 (P/A, 12%, 15) – 4000 (P/G, 12%, 15)

= 5, 00,000 * 6.8109 – 4,000*33.9202

= **Rs 32, 69,769.2 (Ans)**

4. Maintenance cost for a new communication tower with an expected 50 years life are estimated to be Rs 100,000 each years for the first 5 years, followed by a Rs 10,00,000 expenditure in the 15th year and another Rs 10,00,000 expenditure in the 30th year. If MARR = 10%, what is the equivalent uniform annual cost over the entire 50 period? **(PU 2006)**

Solution:



Discounting all the cash flows to the present

PW (10%) = 1, 00,000 (P/A, 10%, 5) + 10, 00,000 (P/F, 10%, 15) + 10, 00,000 (P/F, 10%, 30)

= 1, 00,000 * 3.7908 + 10, 00,000 * 0.2394 + 10, 00,000 * 0.0573

= 3, 79,080 + 2, 39,400 + 57,300

= **Rs 6, 75,780**

Applying the capital recovery factor

AW (10%) = 6, 75,780 (A/P, 10%, 50)

= 6, 75,780 * 0.10086

= **Rs 68,159.17 (Ans)**

5. Find benefit cost ratio by both methods PW and AW method where **(TU, IOE 2064)**

Investment	= Rs 90,000
Annual revenue	= Rs, 50,000
Annual Cost	= Rs 2000
Salvage value	= Rs 20,000
MARR	= 12%
N	= 10 years

Solution

Using PW formulation

PW (12%) $_{\text{net benefits}}$ = 50,000 (P/A, 12%, 10) = 50,000 *5.6502
= Rs 2, 82,510

PW (12%) $_{\text{annual cost}}$ = 2,000 (P/A, 12%, 10) = 2,000 *5.6502
= Rs 11,300.4

PW (12%) $_{\text{salvage value}}$ = 20,000 (P/F, 12%, 10) = 20,000* 0.3220
= Rs 6,440

Conventional B/C ratio $= \dfrac{PW\ (B)}{(I) - PW\ (S) + PW\ (O\&M)}$

$= \dfrac{282510}{90{,}000 - 6{,}440 + 11{,}300.4}$

= 2.98 > 1 (Justified)

Modified B/C ratio $= \dfrac{PW\ (B) - PW\ (O\&M)}{(I) - PW\ (S)}$

$= \dfrac{2{,}82{,}510 - 11{,}300.4}{90{,}000 - 6{,}440}$

= 3.25>1 (justified)

Using AW formulation

Conventional B/C ratio $= \dfrac{AW\ (B)}{CR + AW\ (O\&M)}$

$= \dfrac{50{,}000}{\{90{,}000\ (A/P, 12\%, 10) - 20{,}000\ (A/F, 12\%, 10)\} + 2{,}000}$

= 2.97 > 1 (Justified)

Modified B/C ratio $= \dfrac{AW\ (B) - AW\ (O\&M)}{(CR)}$

$= \dfrac{50{,}000 - 2{,}000}{14{,}788.6}$

= 3.25 >1 (justified)

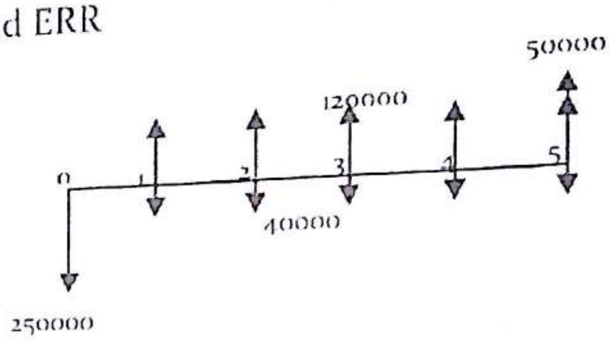
6. Equipment costs 2. 50,000 and has salvage value of 50,000 at the end of its expected life 5 years. Annual expenses will be 40,000. It will produce revenue of 120,000 per year. MARR = 20%=ε

(i) Evaluate IRR using AW formulation

(ii) Evaluate both types of B/C ratio with FW formulation

(iii)Find ERR

Solution



(i) Calculation of IRR

AW (i* %) = 0

$AW_{inflow} - AW_{outflow} = 0$

120000 + 50000 (A/F, i*%, 5) -40000 – 250000 (A/P,i* %,5) =0

120000 + 500000 { i*/ $(1+i^*)^5$ -1}- 40000 -250000 { ((1+i*)5 *i*)/ ($(1+i^*)^5$ -1)}

Solving by hit and trial (calculator)

We get, i* % = 0.21577 =21.577%

IRR = 21.577% (Ans)

(ii) Finding both B/C ratio using FW formulation

$$\text{Conventional B/C ratio} = \frac{FW(B)}{FW(I) - (S) + FW(O\&M)}$$

$$\text{Modified B/C ratio} = \frac{FW(B) - FW(O\&M)}{FW(I) - (S)}$$

FW (B) = 120,000 (F/A, 20%, 5) = 120,000* 7.4416 = 892992

FW (I) = 250,000 (F/P, 20%, 5) = 250,000 * 2.4883 = 622075

FW (O&M) = 40,000 (F/A, 20%, 5) = 40,000 * 7.4416 =297664

Conventional B/C = $\frac{892992}{622075 - 50000 + 297664}$

= 1.026 > 1 (justified)

Modified B/C = $\frac{892992 - 297664}{622075 - 50000}$

= 1.04 > 1 (justified)

(iii) Calculating ERR

Step 1: Discount all the cash outflow to year zero at ε%

= 250000 + 40000 (P/A, 20%, 5)

= 250000 + 40000 * 2.9906 = 369624

Step II: Compound all the cash inflow to period N at ε%

= 120000 (F/A, 20%, 5) + 50,000

= 120000* 7.4416 + 50,000 = 942992

Step III: Establishing the equivalence between above two equations

369624 (F/P,i', 5) = 942992

Or, 369624 $(1+ i')^5$ = 942992

Or, $(1+ i')^5$ = 2.5512

Or, (1+ i') = 1.206

Or, i' = 1.206 -1 =0.206

Or, i'=20.6 %

External rate of Return (ERR) = **20.6%** (Ans)

6. For the cash flow given below, compute Payback Period, Net annual worth, IRR and BCR if MARR is 12%

Solution

Using payback Period.

Period	Cash	PW at 12%	Cumulative

	Flow		Cash flow
0	-25,000	-25,000	-25,000
1	10,000	$10{,}000\,(1+0.12)^{-1} = 8928.57$	- 16,071.4
2	-5,000	$-5{,}000\,(1+0.12)^{-2} = -3985.97$	-20,057.4
3	25,000	$25{,}000\,(1+0.12)^{-3} = 17794.51$	-2,262.89
4	30,000	$30{,}000\,(1+0.12)^{-4} = 19065.54$	16802.65

Here the Cumulative cash flow is negative on year 3 and positive on year 4. Hence the payback period lies between yea 3 and 4. By Interpolating we get,

$3 + 2262.89/19065.54 = 3.118$ years (ans)

Using Net annual worth

We know that, AW (i %) = R – E – CR

Since salvage value is not given, we don't calculate CR

Annual Revenues (R) = {10,000 (P/F, 12%, 1) + 25,000 (P/F, 12%, 3) + 30,000 (P/F, 12%, 4)} (A/P, 12%,4)

= {(10,000 *0.8929) + (25,000 *0.7118) + (30,000 * 0.6355)} * 0.3292

= (8929 + 17795 +19065) * 0.3292

= 45789 *0.3292 = 15073.74

Annual Expenses (E) = 5000 (P/F, 12%,2) (A/P,12%,4)

= 3985.97 * 0.3292 = 1312.18

AW (12%) = 15073.74 - 1312.18 = **13761.56**

Using IRR method

Using PW formulation

PW (i* %) = 0

$PW_{inflow} - PW_{outflow} = 0$

10,000 (P/F, 12%, 1) + 25,000 (P/F, 12%, 3) + 30,000 (P/F, 12%, 4) = 0

$10{,}000\,(1+i^*)^{-1} + 25{,}000\,(1+i^*)^{-3} + 30{,}000\,(1+i^*)^{-4} - 5{,}000\,(1+0.12)^{-2} - 25{,}000 = 0$

Solving by trial and error,

We get i*% = 32.827%, hence IRR = 32.827 %(Ans)

Using BCR method

Using PW formulation

BCR = PW (B) / PW (C)

= {10,000 (P/F, 12%, 1) + 25,000 (P/F, 12%, 3) + 30,000 (P/F, 12%, 4) } / 5000 (P/F, 12%,2) + 25,000

= (8929 + 17795 + 19065) / 1312.18 +25,000

= 45789 / 26312.18

= 1.740 >1 (Justified)

Review Questions

1. What are the methodologies for evaluating the project? Explain briefly
2. Explain present worth, future worth and annual worth?
3. What do you understand by capital recovery? Explain.
4. Define IRR and explain its drawbacks.
5. What is External rate of return (ERR) and what are its advantages over IRR.
6. What are simple and non simple investments? Explain.
7. Explain simple and discounted payback period method. List down its advantages and disadvantages.
8. What do you understand by benefit/cost analysis? Explain its types.
9. What do you understand by public sector project?

Exercises

Equivalent worth Method

10. Consider the following set of investment projects. All projects have 3 year investment life. Compute the Present worth and Future Worth of each project. MARR =10%.

Project's cash flow (Rs)				
N	A	B	C	D
0	-1000	-1000	-1000	-1000
1	0	600	1200	900
2	0	800	800	900
3	3000	1500	1500	1800

STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- To understand the concept of replacement analysis
- To understand the factors considered in replacement analysis
- To determine the economic service life of challenger and defender
- To analyze the replacement problem when required service life is infinite.
- To analyze the replacement problem when required service life is finite.

5.1 Introduction

- When should the new machine replace the existing machine?
- When should a process be redesigned?
- When should a product be redesigned?

The most common question asked in the industry or organization is when should the existing be replaced?

Replacement analysis is the economic analysis to compare existing and new facilities. It is a decision situation encountered in the business firms, and government organizations as well as individuals in which an existing asset should be retired from use or continued in service or replaced with a new asset. Replacement study in engineering economics provides the information for sound decisions that improve the operating efficiency and the competitive position of enterprise. Replacement analysis is one of the most important and most common types of iterative comparisons encountered in practice. In some organizations, replacement analysis is performed routinely in an effort to ensure that the best equipment and facilities are in use, compared to their possible successors.

The reasons for considering replacement are numerous. Firstl the current asset (defender) may have number of deficiencie including high set-up cost; excessive maintenance, declinin production efficiency energy consumption, and physica impairment. For example, when you are confronted with a ca that is expensive to operate and maintain, and a major overhaul, you consider replacing the car.

Secondly, potential replacement assets (challengers) may take advantage of new technology and be easily set up, maintained at low cost, high in output, energy efficient and possessing increased capabilities, perhaps at a vastly reduced cost. For example, some new generation computer-controlled manufacturing equipment has rendered many old machines economically obsolete. Also, we can relate to the phenomenal accomplishments with which calculators and personal computers have resulted in increased capabilities, vastly lower prices, and economic obsolescence for equipment only a few years old.

Replacement is never a question of "if we replace" but rather a question of "when we replace". The key question is: "Shall we replace the defender now, or keep it for one or more years before replacing it?"

Replacement of an existing asset should be considered in case of following reasons:

- **Obsolescence:** occurs when the technology of an asset is surpassed by newer and or different technologies.
- **Depletion:** the gradual loss of market value of an asset as it is being consumed or exhausted.
- **Deterioration due to aging:** the general condition of loss in value of some asset due to aging process. The aging process is normally associated with additional maintenance and operating expenses.

- **Physical Impairment:** The existing equipment is completely or partially worn out and will no longer function satisfactorily without expensive repairs.
- **Inadequacy**: The equipment does not have sufficient capacity to meet the present demands.
- **Rapid Technological Changes:** Recognition and handling of replacement problems have paid off quite well in many companies. Some of the advantages are:

(a) Maintenance costs would be reduced.

(b) Production costs would be reduced and would keep the company competitive.

(c) Losses, scraps, rework would be reduced.

(d) Modernization would be introduced which will help to take-off' productivity and returns,

(e) Delays off down-time costs would be reduced.

(f) Enthusiasm and morale of workers would be increased resulting into increased human efficiency, better human relations.

It is clear from these points that replacement should be based on economy, since it is an economic venture which must yield considerable profits. Replacement also adds modernization which is essential for growth. Some other reasons which have already been discussed above may include inadequacy, excessive maintenance, decline of efficiency, obsolescence etc.

There are three basic ingredients to the successful handling of replacement problems.

(i)There must be clearly stated policies to guide the persons handling the replacement problems.

(ii)The replacement problems must be recognized in the organization structure by specific assignment of responsibility.

(iii)A systematic procedure must be established and used in solving specific problems.

Terminologies

1. **Defender and Challenger:** These are the names of two mutually exclusive alternatives. The defender is the currently installed asset and the challenger is the potential replacement asset.

2. **Defender first cost:** It is the initial investment amount P used for the defender. The current market value (MV) is the correct estimate to use for P for the defender in a replacement study.

3. **Challenger first cost:** It is the amount of cost that must be recovered when replacing a defender with challenger. This amount is almost always equal to P, the first cost of challenger.

4. **AW Values:** These are used as the primary measure of comparison between the defender and challenger.

5. **Economic Life:** The period of time (years) that results in the minimum equivalent uniform annual cost of owning and operating an asset.

6. **Useful Life:** The time period (years) that an asset is kept in productive service. It is estimate of how long an asset is expected to be used in a trade or business to produce income.

7. **Marginal Cost:** Marginal costs are the year by year estimates of the costs to own and operate an asset for that year. It includes loss in value of the asset by

retaining it for one or more year, cost and expenses directly related to the project or asset (insurance, operating and maintenance etc).

8. **Market Value:** The highest estimated price that a buyer would pay and a seller would accept for an item in an open and competitive market.

5.2 Approach for comparing Defender and Challenger

1. **Cash Flow Approach**

In a cash flow approach, proceeding from the sale of the old machine is treated as down payment towards purchasing the new machine. This approach is meaningful when both the defender and challenger have same useful life. Net present worth method and annual equivalent worth method is used for the comparison.

2. **Opportunity Cost Approach**

In an opportunity cost approach, proceedings from the sale of old machine is treated as the investment required to keep the old machine. This approach is more commonly practiced in replacement analysis.

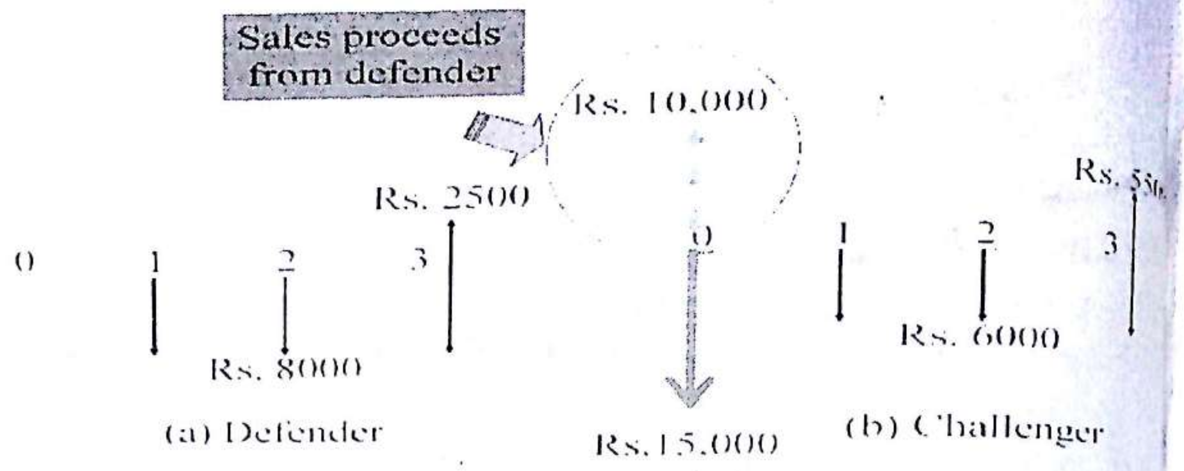
Example 5.1

Decide whether replacement is justified for

Defender	Challenger
Market Price = Rs 10,000	Initial Cost = Rs 15,000
Remaining Useful life = 3 years	Useful Life = 3 Years
Salvage Value = Rs 2500	Salvage Value = Rs 5,500
Operation and Maintenance cost = Rs 8000	Operation and Maintenance = Rs 6,000
MARR = 12%	

Solution:

(a) From Cash Flow Approach



Defender:

PW (12%) $_D$ = Rs.2, 500 $(P/F, 12\%, 3)$ – Rs.8, 000 $(P/A, 12\%, 3)$

= - Rs.17, 434.90

AW (12%) $_D$ = PW (12%) $_D$ $(A/P, 12\%, 3)$

= -Rs7, 259.10

Challenger:

PW (12%) $_C$ = Rs.5, 500 $(P/F, 12\%, 3)$ - Rs.5, 000 - Rs.6, 000 $(P/A, 12\%, 3)$

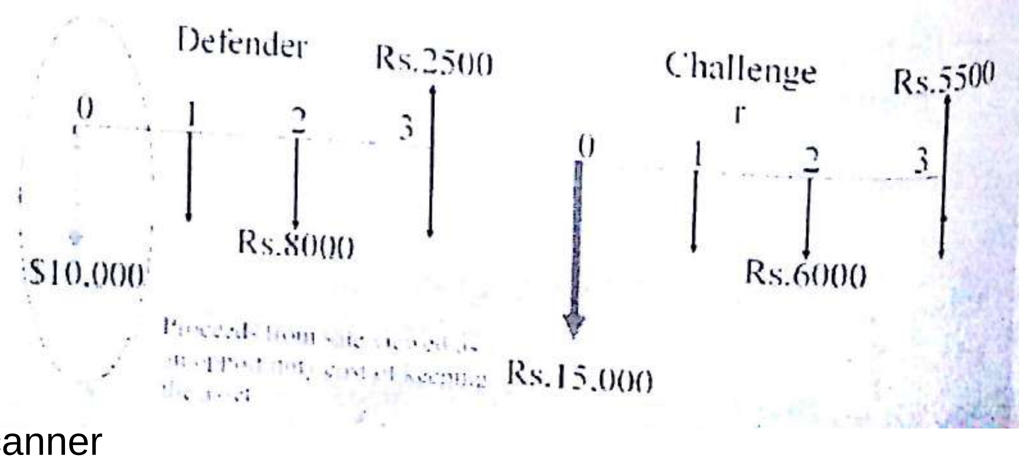
= - Rs.15, 495.90

AW (12%) $_C$ = PW (12%) $_C$ $(A/P, 12\%, 3)$

= - Rs.6, 451.79

Here AW of Challenger is greater than Defender, **replace the defender**.

(a) From Opportunity Cost Approach



Defender:

PW (12%) D = -Rs.10, 000 - \$8,000(P/A, 12%, 3) + Rs.2, 500(P/F, 12%, 3)

= - Rs.27, 434.90

AW (12%) D = PW (12%) D (A/P, 12%, 3)

= - Rs.11, 422.64

Challenger:

PW (12%) C = - Rs.15, 000 - Rs.6, 000(P/A, 12%, 3) + Rs.5, 500(P/F, 12%, 3)

= - Rs.25, 495.90

AW (12%) C = PW (12%) C (A/P, 12%, 3)

= - Rs.10, 615.33

Here AW of Challenger < AW of Defender, replace the defender

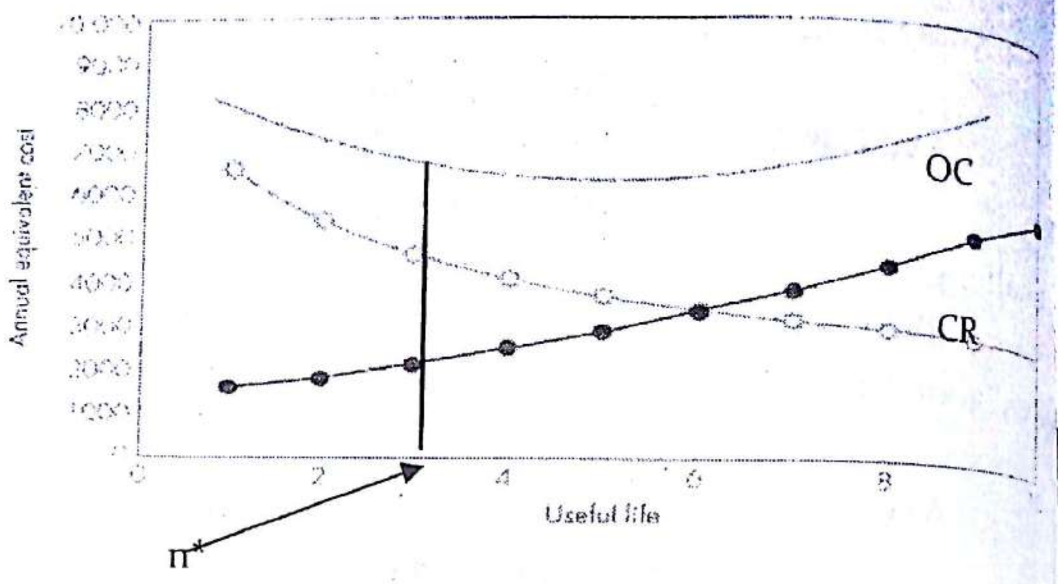
5.3 Economic Service Life (ESL)

The economic service life (ESL) is the number of year (n*) at which the equivalent uniform annual worth (AW) of costs is the minimum, considering the most current cost estimates over all possible years that the asset may provide a needed service. We should use the respective economic service lives of the defender and the challenger when conducting a replacement analysis.

Mathematical Relationship

The objective is to find the number of year (n*) that minimizes the equivalent uniform annual worth of costs, is minimum.

- $Capital\ Recovery\ Cost\ (CR) = I\left(\frac{A}{P}, i\%, N\right) - S\left(\frac{A}{F}, i\%, N\right)$
- $Operating\ Cost\ (OC) = \sum_{n=1}^{N} OCn\left(\frac{P}{F}, i\%, n\right)\left(\frac{A}{P}, i\%, N\right)$
- $Total\ Cost = CR\ (i) + OC\ (i)$



5.4 Minimum Cost life of the Challenger

Minimum cost life of any new (existing) asset is the number of years at which the equivalent uniform annual cost (EUAC) of the ownership is minimized. That is we need to identify the year in which the total EUAC is minimized. The minimum cost life is normally shorter than the useful life of the asset because of increasing operating and maintenance cost in the later years of asset life.

The minimum cost life is calculated as the following:

- Calculate the EUAC for each value of the useful life (e.g., n = 1, n = 2, n = 3, etc.)
- The EUAC decreases to some minimum and increases again.
- The minimum cost life is the number of years at which the EUAC is minimum.
- The asset having the lowest minimum cost life of all the challengers is chosen to compete with the defender asset.

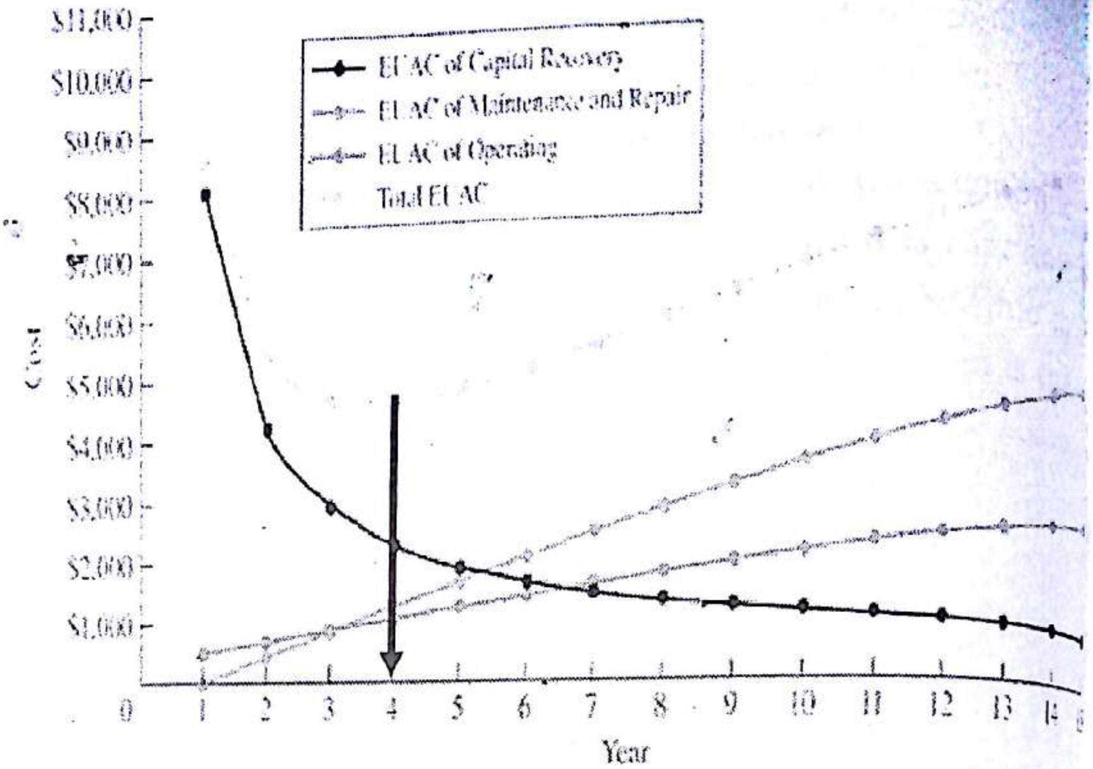
Example 5.2

A piece of machinery costs Rs.7500 and has no salvage value after it is installed. No maintenance or repair is required in the first year. The second year, the maintenance cost will be Rs 900 and will increase on an Rs.900 arithmetic gradient in the subsequent years. The first year operating expense will be Rs.500 and will increase on an Rs.400 arithmetic gradient in the following years. If interest is 8%, compute the useful life of the machinery that will result in a minimum EUAC (Find its minimum cost life).

Solution:

Year	EUAC of Capital Recovery Rs 7500 (A/P,8%,n)	EUAC of Maintenance and Repair Cost Rs 900 (A/G, 8%,n)	EUAC of Operating Costs Rs 500 + Rs 400 (A/G,8%,n)	Total EUAC (Rs)
1	8100	0	500	8600
2	4205.77	432.69	692.31	5330.77
3	2910.25	853.87	879.50	4643.62
4	**2264.41**	**1263.56**	**1061.58**	**4589.55**
5	1878.42	1661.82	1238.59	4778.84
6	1622.37	2048.71	1410.54	5081.62
7	1440.54	2424.30	1577.47	5442.31
8	1305.11	2788.67	1739.41	5883.19
9	1200.60	3141.93	1896.41	6238.94
10	1117.72	3484.18	2048.53	6650.43
11	1050.57	3815.55	2195.80	7061.93
12	995.21	4136.17	2338.30	7469.68
13	948.91	4446.19	2476.08	7871.18
14	909.73	4745.75	2609.22	8264.69
15	876.22	5035.01	2737.78	8649.02

The EUAC are estimated for 15 years period and plotted in the following figure. From either the table or the figure, the minimum cost life of the machinery is 4 years, with minimum EUAC of Rs.4589 for each of those 4 years.



Economic life is where the Total EUAC is minimized.

Example 5.3

A chemical plant owns a filter press that was bought 3 years ago for Rs.30, 000. Now it has a market value of Rs.9000, a life of 5 years, and a salvage value of Rs.2000 at that time. The challenger has a cost of Rs.36, 000, a life of 10 years, and estimated market value of Rs.12, 000 after 5 years. 5 years planning period to be used and the MARR is 15%. The operating and maintenance cost of the alternatives are given in the following table:

End of Year (n)	Alternative 1 Operation and Maintenance Cost (Rs)	Alternative 2 Operation and Maintenance Cost (Rs)
0	0	0
1	7000	0
2	8000	1000
3	9000	2000
4	10000	3000
5	11000	4000

Should the old filter press be replaced now?

Solution:

Cash flow for replacement (opportunity cost approach)

End of Year (n)	Alternative 1	Alternative 2
0	-9000	-36000
1	-7000	0
2	-8000	-1000
3	-9000	-2000
4	-10000	-3000
5	-11000 +2000	-4000 +12000

$EUAW_1$ = - Rs.9000 (A/P, 15, 5) - Rs.7000 - Rs.1000 (A/G, 15, 5) + Rs.2000 (A/F, 15, 5)

= - Rs.11, 110.90 / year

$EUAW_2$ = - Rs.36, 000(A/P, 15, 5) - Rs.1000 (A/G, 15, 5) + Rs.12, 000(A/F, 15, 5)

= - Rs.10, 682.00 / year

The difference in annual worth is Rs.428.90 / year in favor of the new filter press.

Alternatively

$NPW_1 = -9{,}000 - 7{,}000\,(1+0.15)^{-1} - 8{,}000\,(i+0.15)^{-2} - 9{,}000\,(1+0.15)^{-3} - 10{,}000\,(1+0.15)^{-4} - 9{,}000\,(1+0.15)^{-5}$

$= -37245.9$

$NPW_2 = -36{,}000 + 0 - 1{,}000\,(1+0.15)^{-2} - 2{,}000\,(1+0.15)^{-3} - 3{,}000\,(1+0.15)^{-4} + 8{,}000\,(1+0.15)^{-5}$

$= -20293.8$

NPW_2 (-ve) < NPW_1 (-ve), alternative 2 is better than alternative 1 i.e. challenger is better than defender, so the old filter press should be replaced.

5.5 Replacement Analysis when Required Service Life is long.

Required Assumption and Decision Frame Work

In previous section, it is understood that how the economic service life of an asset is determined. The question is that how to use this information to decide whether now is the time to replace the defender. If now is not the right time, when is the optimal time to replace the defender? For answering the above question the following three assumptions are made.

1. Planning horizon (study period)
2. Technology
3. Relevant cash flow information

1. Planning horizon

By planning horizon it is mean that the service period required by the defender and a sequence of future challenger.

(a) *Infinite planning horizon*: It is used when we are unable to predict when the activity under consideration will be terminated.

(b) *Finite planning horizon:* When the project has a definite and predictable duration, replacement policy should be formulated more realistically based on a finite planning horizon.

2. Technology

Prediction of technological patterns over the planning horizon refers to the development of types of challengers that may replace those under study. A number of possibilities exist in predicting purchase cost, salvage value and operating cost dictated by the efficiency of machine over the life of an asset. If we assume that all future machines will be same as those now in service, there is no technological progress in the area will occur. In other cases, we may recognize the possibility of future machine that will be significantly more efficient, reliable or productive than those currently on the market.

3. **Relevant cash flow information**

Many varieties of prediction can be used to estimate the patterns of revenue, cost, and salvage value over the life of an asset. Sometimes revenue is constant, but costs increase, while salvage value does not, over the life of the machine. In other situations, a decline in revenue over equipment life can be expected. The specific situation will determine whether replacement analysis is directed toward cost minimization (with constant revenue) or profit maximization (with varying revenue).

Decision Frameworks

The annual worth / annual equivalent (AE) method provides a more direct solution when the planning horizon is infinite (endless). Similarly when the planning horizon is finite (fixed) the present worth (PW) method is convenient to be used.

5.6 Replacement Analysis under Infinite Planning Horizon

Under the infinite planning horizon, the service is required for a very long time. Either we continue to use the defender to provide the service or we replace the defender with the best available challenger for the same service requirement

Procedure for replacement analysis (infinite planning horizon)

- Compute the economic service life of the both defender and challenger. Use N_D * (economic service life of defender) and N_C* (economic service life of Challenger) respectively. The annual equivalent cost for the defender and challenger at their economic lives are indicted by AE_D* and AE_C*.
- Compare AE_D* and AE_C*. If AE_D* is bigger than AE_C*, we know that it is more costly to keep the defender

than to replace it with the challenger. Thus the challenger should replace the defender now.

- If the defender should not be replaced now, when should it be replaced? First, we need to continue to use until its economic service life is over. Then we should calculate the cost of running the defender for one more year after its economic life. If this cost is greater than AE_C^*, the defender should be replaced at the end of the its economic life . this process should be continued until we find the optimal replacement time. This approach is called the *marginal analysis, i.e.* to calculate the incremental cost of operating the defender for just one more year.

Example 5.4

A company is considering the replacement of the old inspection machine. If the machine is repaired, it can be used for 5 more years. It can be sold to the other firm in Rs. 5,000. If the machine is kept it will require an immediate overhaul (renovation) of Rs. 1200 to make it in operable condition. Overhaul charge is not extended for service life. The operation costs are estimated at Rs 2000 during first year and these are expected to increase by Rs. 1500 per year thereafter. Future market values are expected to decline by Rs.1000 per year.

The new machine costs Rs 10,000 and will have operating costs of Rs. 2000 in the first year, increasing Rs 800 per year thereafter. Salvage value is Rs 6000 after one year and will decline by 15% each year. The company requires a rate of return of 15%. Determine Economic life of each option and when the defender should be replaced?

Solution

Relevant cash flow information for defender

n	Overhaul (Rs)	Operating Cost (Rs)	Salvage Value (Rs)
0	1200	0	5000
1	0	2000	4000
2	0	3500	3000
3	0	5000	2000
4	0	6500	1000
5	0	8000	0

When n=1

CR (15%) = 6200 (A/P, 15%, 1) – 4000 (A/F, 15%, 1)

= 6200 *1.15 – 4000 *1 = 3,130.

Annual equivalent cost $(AEC)_1$ = 3130 + 2000 =5,130

When n=2

CR (15%) = 6200 (A/P, 15%, 2) – 3000 (A/F, 15%, 2)

= 6200 *0.6151 – 3000 *0.4631= 2,418.32

Operating Cost = {2000 (P/F, 15%, 1) + 3500 (P/F, 15%, 2)} (A/P, 15%, 2)

= 2000*0.8696 + 3500 * 0.7561} * 0.6151

= 2697.55

Annual equivalent cost $(AEC)_2$ = 2,418.32 + 2697.55 =5,115. 87

When n=3

CR (15%) = 6200 (A/P, 15%, 3) – 2000 (A/F, 15%, 3)

= 6200 *0.4380 – 3000 *0.2880= 2,139.60

Operating Cost = {2000 (P/F, 15%, 1) + 3500 (P/F, 15%, 2) + 5000 (P/F, 15%, 3)} (A/P, 15%, 3)

= 2000*0.8696 + 3500 * 0.7561 + 5000 * 0.6575} * 0.4379

= 3360.02

Annual equivalent cost $(AEC)_3$ = 2,139.60 + 3360.02 =5,499.82

When n=4

CR (15%) = 6200 (A/P, 15%, 4) – 1000 (A/F, 15%, 4)

= 6200 *0.3503 – 3000 *0.2003= 1,972.56

Operating Cost = {2000 (P/F, 15%, 1) + 3500 (P/F, 15%, 2) + 5000 (P/F, 15%, 3) + 6500 (P/F, 15%, 4)} (A/P, 15%, 4)

= 2000*0.8696 + 3500 * 0.7561 + 5000 * 0.6575 +6500* 0.5718} * 0.3503

= 3989.43

Annual equivalent cost $(AEC)_4$ = 1,972.56 + 3989.43= 5,960.99

When n=5

CR (15%) = 6200 (A/P, 15%, 4) – 0

= 6200 *0.3503 = 1,850

Operating Cost = {2000 (P/F, 15%, 1) + 3500 (P/F, 15%, 2) + 5000 (P/F, 15%, 3) + 6500 (P/F, 15%, 4+8000 (P/F, 15%, 5)} (A/P, 15%, 5)

= 2000*0.8696 + 3500 * 0.7561 + 5000 * 0.6575 +6500* 0.5718 +0.4972} * 0.2983

= 4584

Annual equivalent cost $(AEC)_5$ = 1,850 + 4584= 6,434

From year one to five, the annual equivalent cost of defender is as follows:

Year (n)	AEC (Rs)
1	5,130
2	5,115.87
3	5,499.62
4	5,960.99
5	6,434

When n= 2 years, we can see the lowest AEC value. Thus the defender's economic service life is 2 years. We now determine the economic service life of the challenger.

We have, Investment = Rs 10,000,

Salvage Value at year 1 = Rs 6000 and decreases at 15% over previous year.

Operating Cost = Rs 2000 and increases by Rs 800 per year

MARR = 15%

When n =1,

CR (15%) = 10,000 (A/P, 15%, 1) – 6000 (A/F, 15%, 1)

= 10,000 * 1.15 – 6000 * 1 = 5,500

Operating Cost = 2,000

Annual equivalent cost $(AEC)_1$ = 5,500 + 2,000 = 7,500

When n =2,

CR (15%) = 10,000 (A/P, 15%, 2) – 5100 (A/F, 15%, 2)

= 10,000 * 0.6151 – 5100 * 0.4651 = 3,779

Operating Cost = {2000 (P/F, 15%,1) + 2,800 (P/F, 15%,2) } (A/P,15%,2)

= 2000* 0.8696 + 2800 * 0.7561} * 0.6151

= 2372

Annual equivalent cost $(AEC)_2$ = 3,779 + 2,372 = 6,151

When n =3,

CR (15%) = 10,000 (A/P, 15%, 3) – 4335 (A/F, 15%, 3).

= 10,000 * 0.4380 – 4335 * 0.2880 = 3,132

Operating Cost = {2000 (P/F, 15%, 1) + 2,800 (P/F, 15%, 2) + 3600 (p/f, 15%, 3)} (A/P, 15%, 3)

= 2000* 0.8696 + 2800 * 0.7561+ 3600 *0.6575} * 0.4380

= 2725

Annual equivalent cost (AEC) $_3$ = 3,132 + 2,725 = 5,857

When n = 4,

CR (15%) = 10,000 (A/P, 15%, 4) – 3685 (A/F, 15%, 4)

= 10,000 * 0.3503 – 3685* 0.2003 = 2,765

Operating Cost = {2000 (P/F, 15%, 1) + 2,800 (P/F, 15%, 2) + 3600 (P/F, 15%, 3) + 4400 (P/F, 15%, 4)} (A/P, 15%, 3)

= 2000* 0.8696 + 2800 * 0.7561+ 3600 *0.6575 + 4400 * 0.5718} * 0.3503

= 3061

Annual equivalent cost (AEC) $_4$ = 2,765 + 3,061 = 5,826

When n = 5,

CR (15%) = 10,000 (A/P, 15%, 5) – 3132 (A/F, 15%, 5)

= 10,000 * 0.2983 – 3132* 0.1483 = 2,519

Operating Cost = {2000 (P/F, 15%, 1) + 2,800 (P/F, 15%, 2) + 3600 (P/F, 15%, 3) + 4400 (P/F, 15%, 4)+ 5200 (P/F, 15%, 5)} (A/P,15%,5)

= 2000* 0.8696 + 2800 * 0.7561+ 3600 *0.6575 + 4400 * 0.5718 + 5200 * 0.4972} * 0.2983

= 3378

Annual equivalent cost (AEC) $_5$ = 2,519 + 3,378 = 5,897

From year one to five, the annual equivalent cost of challenger is as follows:

Year (n)	AEC (Rs)
1	7,500
2	6,151
3	5,857
4	5,826
5	5,897

When n= 4 years, we can see the lowest AEC value. Thus the defender's economic service life is 4 years.

Replacement Decision

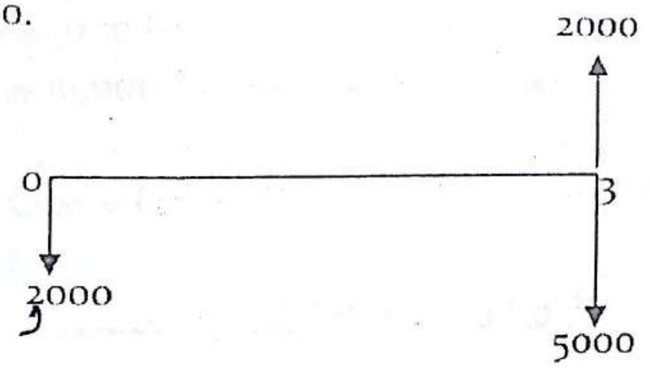
The annual equivalent cost (AEC) of defender (5116) < The annual equivalent cost of Challenger (5826). Hence the defender is not replaced now. If there is no technological advancement in the next few years the defender should be used for at least 2 more years.

We have to calculate the cost of keeping and using defender for the third year from today. That is not selling the defender at the end of year 2 using it for the third year and replacing it at the end of year 3.

(a) Opportunity cost at the end of year 2 = Rs. 3000 (market value of the defender at the end of year 2)

(b) Operating cost for 3^{rd} year : Rs. 5000

(c) Salvage value of the defender at the end of year 3 = Rs. 2000.



- Calculate the equivalent cost of the defender one more year from the end of its economic service life.
  3000 (F/P, 15%, 1) + 5000 -2000 = Rs. 6450
- Compare this cost with annual equivalent cost of challenger (AEC) = Rs. 5826.

Since the equivalent cost of the defender is greater than annual equivalent cost of the challenger, it is concluded that the defender should be replaced at the end of year 2.

Example 5.5

Three years ago Chicago O'hare Airport purchased a new fire truck. Because of flight increases, new fire-fighting capacity is needed once again. An additional truck of the same capacity can be purchased now, or a double capacity truck can replace the current fire truck. Estimates are presented below. Compare the options at 12% per using (a) a 12-year study period.

Year	Presently owned	New Purchase	Double Capacity
First Cost (Rs)	-151000 (3 years ago)	-175000	-190000
AOC (Rs)	-1,500	-1,500	-2,500
Marked Value (Rs)	70,000	-	-
Salvage Value (Rs)	10%of first cost	12%of first cost	10%of first cost
Life in Years	12	12	12

Solution

- Identify option 1 as retention of the presently owned truck and augmentation (expansion) with a new same capacity vehicle.
- Define option 2 as replacement with the double capacity truck.

	Option1		Option 2
	Presently owned	Augmentation	Double Capacity
First Cost	-70,000	-175,000	-190,000
AOC	-1,500	-1,500	-2,500
Salvage value	15,100	21,000	19,000
Life	9 yrs	12 yrs	12 yrs

(a) For a full-life 12-year study period of option 1

AW_1 = (AW of presently owned) + (AW of Augmentation)

= [-70,000 (A/P,12%, 9) + 15,100 (A/F, 12%,9) -1500] + [-175,000(A/P,12%,12) + 21,000 (A/F, 12%,12) -1500)

= -13616 -28,882

= Rs -42,498

(b) For a full-life 12-year study period of option 2

AW_2 = -190,000 (A/P, 12%, 12) + 19,000 (A/F, 12%, 12) -2500

= Rs -32,386

Replace now with the double-capacity truck (option 2) at an advantage of $10,112 per year

5.7 Replacement Analysis under Finite Planning Horizon

If the planning period is finite, comparison based on the annual equivalent cost (AEC) method over a defender's economic service life does not generally apply. The procedure for solving such a problem with a finite planning horizon is to establish all 'reasonable' replacement patterns and then use the equivalent worth value for the planning period to select the most economical pattern.

Example 5.6

An existing machine has a market value of Rs. 50,000 and can be used for six years after repair. The machine will require an immediate Rs 12000 overhaul to restore it to operable condition. The opportunity costs are estimated Rs 20,000 during first year and expected to increase by Rs 15,000 per year thereafter. Book value of the machine is expected to decline by 25% each year over the previous year's value.

New machine can be bought at Rs 1, 00,000 and will have operating cost of Rs 22,000 in the first year and increases by 20% per year thereafter. Book value will be Rs 60,000 after one year and will decline 15% each year. This new machine will last for eight years. Find the economic service life of both the alternatives and determine the best replacement strategy if a firm has a contract to perform a given service for next eight years. MARR =15%

Solution

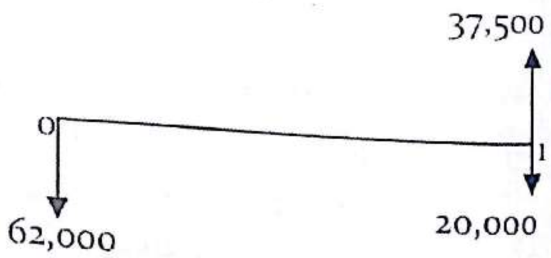
If the existing machine (defender) is retained, total cost will be equal to Rs 50,000 + Rs 12,000 = Rs 62, 000 i.e. Rs 50,000 as opportunity cost and Rs 12,000 as overhaul cost. Operating costs and book values can be calculated as following:

Year	O&M (increasing by Rs. 15,000 per year)	Book Values
1	20,000	75% of 50,000 = 37,500
2	35,000	75% of 37,500 = 28,130
3	50,000	75% of 28,130 = 21,090
4	65,000	75% of 21,090 = 15,820
5	80,000	75% of 15,820 = 11870
6	95,000	75% of 11870 = 8,900

AEC for N=1

$62{,}000 \times 1.15 + 20{,}000 - 37{,}500$

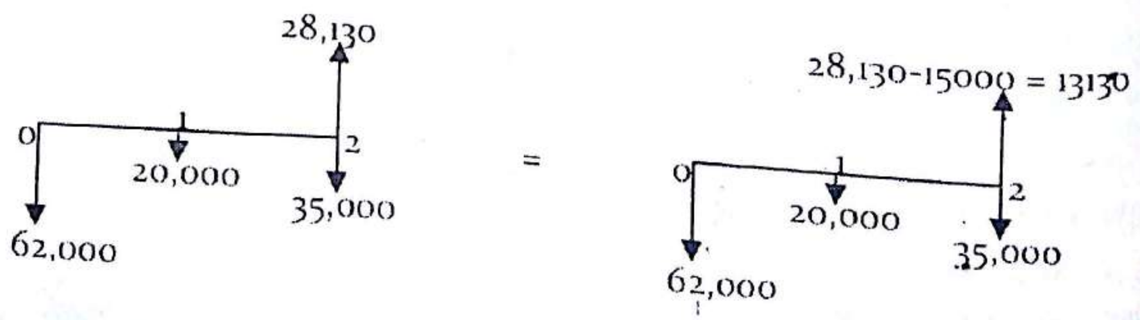
= Rs 53,800



AEC for N=2

$60{,}000\,(A/P, 15\%, 2) + 20{,}000 - 13{,}130\,(A/F, 15\%, 2)$

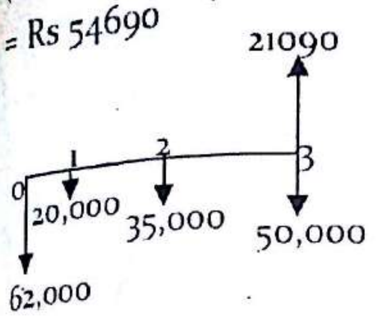
= Rs 52,030



AEC for N=3

$62{,}000\,(A/P, 15\%, 3) + 20{,}000 + \{15000 * 1.15 + 30{,}000 - 21090\}\,(A/F, 15\%, 3)$

= Rs 54690



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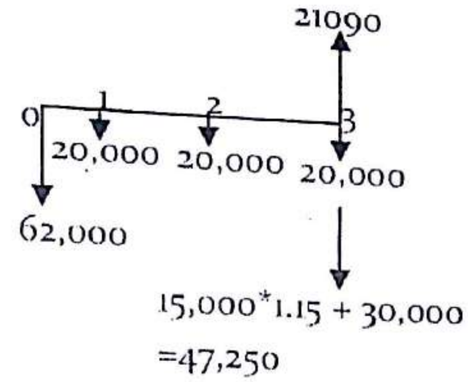
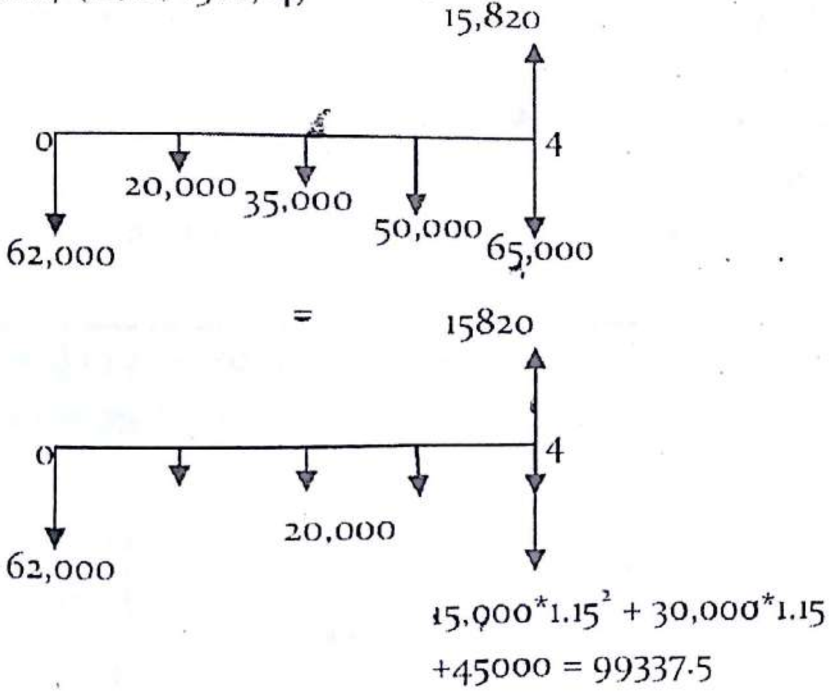


AEC for N=4

$62{,}000\,(A/P, 15\%, 4) + 20{,}000 + \{15000 * 1.15^2 + 30{,}000 * 1.15 + 45{,}000 - 15820\}\,(A/F, 15\%, 4)$

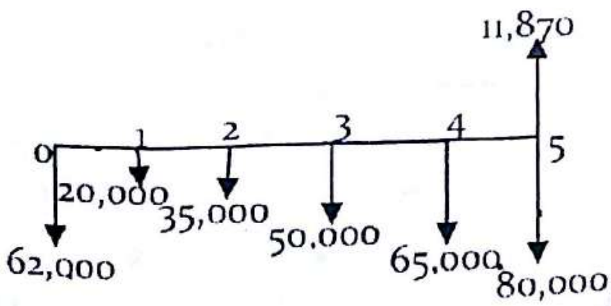
= Rs 58440



AEC for N=5

$62{,}000\,(A/P, 15\%, 5) + 20{,}000 + 15000\,(A/G, 15\%, 5) - 11870\,(A/F, 15\%, 5)$

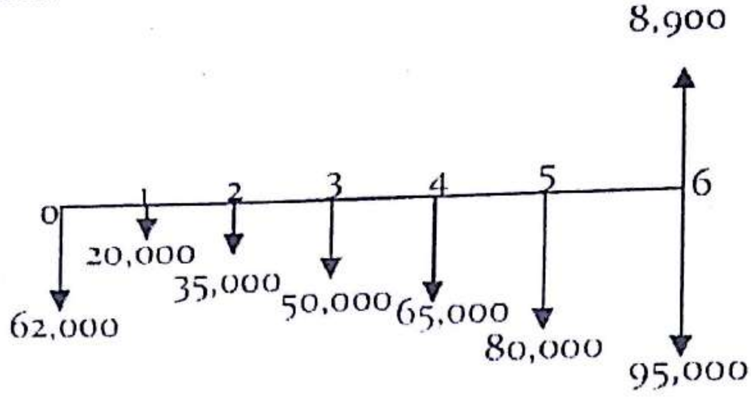
= Rs 62,580



AEC for N=6

62,000 (A/P, 15%, 6) + 20,000 + 15000 (A/G, 15%, 6) -8900(A/F, 15%, 6)

= Rs 66,820



AEC is minimum i.e. Rs 52030 when N =2 years. Thus, the defender's economic service life is 2 years.

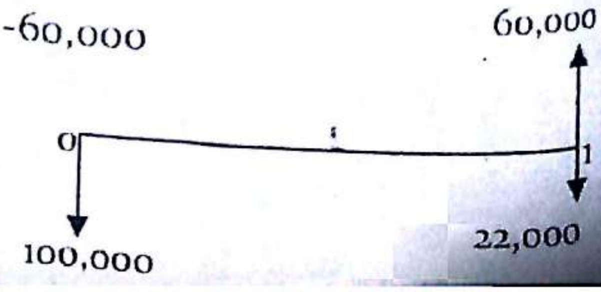
Old machine is replaced by new one (by Challenger), the investment cost will be 100,000 and operation and maintenance cost and book values are as following:

Year	O&M (increasing by 20% per year)	Book Values (decreasing by 15% per year
1	22,000	60,000
2	26,400	51,000
3	31,680	43,350
4	38,020	36,850
5	45,620	31,320
6	54,740	26,620
7	65,690	22,630
8	78,830	19,230

AEC for N=1

100,000 (A/P, 15%, 1) + 22,000 -60,000

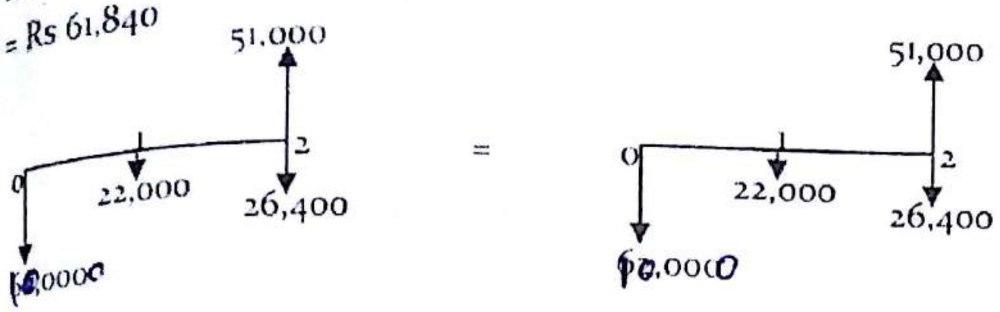
= Rs 53,800



AEC for N=2

100,000 (A/P, 15%, 2) + 22,000 +46,600 (A/F, 15%, 2)

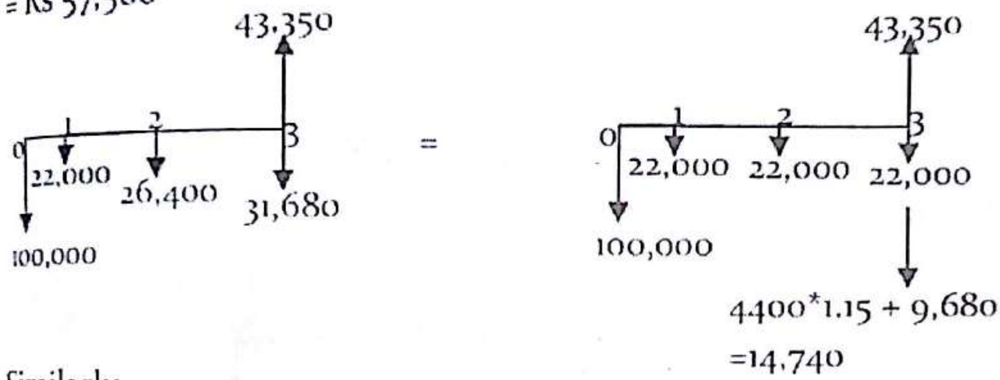
= Rs 61,840



AEC for N=3

100,000 (A/P, 15%, 3) + 22,000 - 28610(A/F, 15%, 3)

= Rs 57,560



Similarly,

AEC_4 = Rs 56,250

AEC_5 = Rs 56,310

AEC_6 = Rs 57210

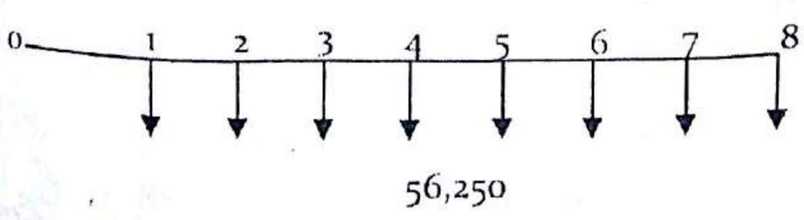
AEC_7 = Rs 58,690

AEC_8 = Rs 60660

Minimum AEC of Challenger is Rs 56250 when N = 4years. Thus, the challenger's economic service life is 4 years.

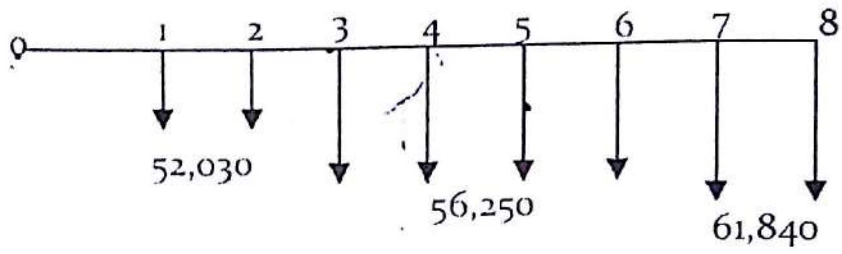
Many replacement scenario options would fulfill an eight year planning horizon as below.

Option 1: Use Challenger for eight years (4 + 4 years)



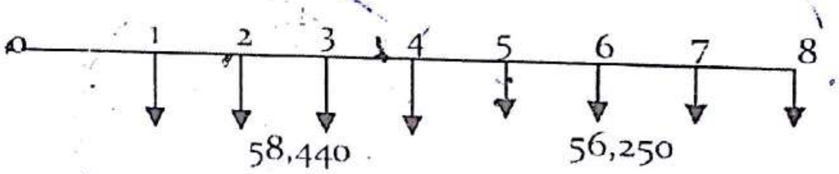
$$PW = 56{,}250\ (P/A,\ 15\%,\ 8) = \text{Rs } 2{,}52{,}410$$

Option 2: Use defender for 2 years, challenger for next four years and challenger for two more years again (2 + 4 + 2)



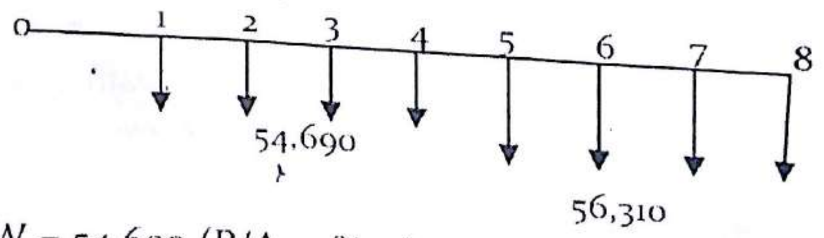
$$PW = 52{,}030\ (P/A,\ 15\%,2) + 56{,}250\ (P/A,15\%,4) * \{P/F,15\%,2\} + 61{,}840\ (P/A,\ 15\%,2) * \{P/F,15\%,6\}$$
$$= 2{,}49{,}480$$

Option 3: Using defender for four years and challenger for remaining four years



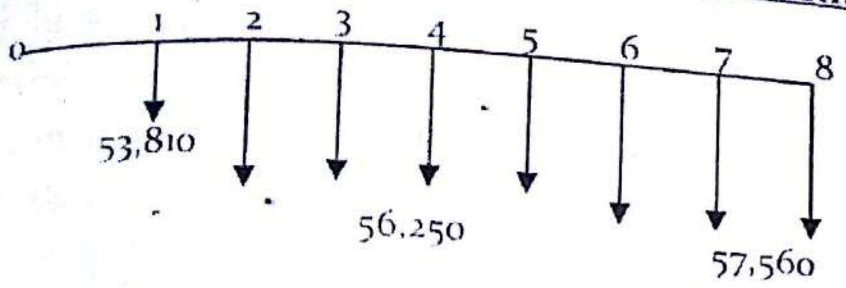
$$PW = 58{,}440\ (P/A,\ 15\%,\ 4) + 56{,}250\ (P/A,\ 15\%,4) * \{P/F,15\%,4\}$$
$$= 2{,}58{,}660$$

Option 4: Using defender for three years and challenger for remaining five years



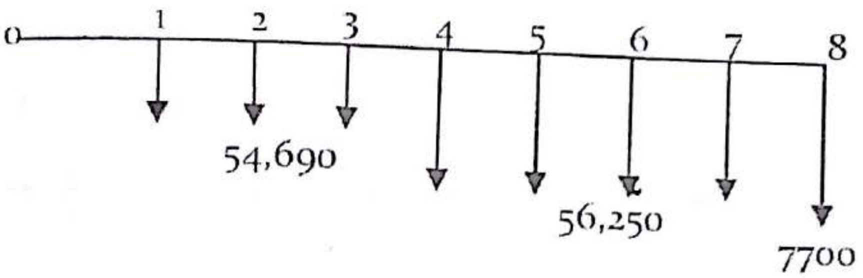
$$PW = 54{,}690\ (P/A,\ 15\%,\ 3) + 56{,}310\ (P/A,\ 15\%,\ 5) * \{P/F,\ 15\%,\ 3\}$$
$$= 2{,}48{,}980 \text{ (minimum)}$$

Option 5: Using defender for only one years and challenger for 7 years (4+3)



$PW = 53{,}800\ (P/A, 15\%, 1) + 56{,}250\ (P/A, 15\%, 4) * \{P/F, 15\%, 1\}$
$+ 57560\ (P/A, 15\%, 4)\ \{P/F, 15\%, 5\}$
$= 2, 51{,}770$

Option 6: Using defender for three years and challenger for 5 years (4+1)



$PW = 54{,}690\ (P/A, 15\%, 3) + 56{,}250\ (P/A, 15\%, 4) * \{P/F, 15\%, 3\}$
$+ 77000\ \{P/F, 15\%, 8\}$
$= 2, 55{,}630$

From above calculations, least cost solution among six decision options considered appear to be option 4 with present worth equivalent of Rs. 2,48,980. So, retain the defender for three years, purchase the challenger and keep it for five years.

Review Question

1. What do you understand by replacement analysis? Explain the reasons for carrying out replacement analysis.
2. Explain the defender, challenger, marginal cost, defender's first cost and challenger's first cost.
3. What is economic service life? Explain with a suitable example.
4. What are the assumptions made in Replacement An...

STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- Basic concept for comparing alternatives.
- The study (analysis) period.
- Comparing the alternatives having useful lives equal to the study period
- Comparing the alternatives having useful lives different and unequal to study period.
- The capitalized worth method.
- To understand the mutually exclusive combinations of the project.

6.1 Introduction

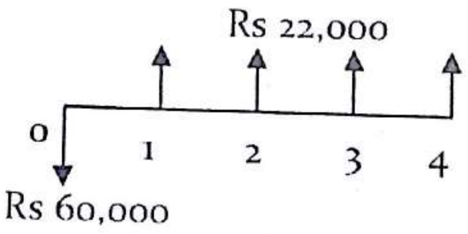
Most engineering projects can be accomplished by more than one feasible design alternative. These projects are funded from the capital budget. When the selection of one of these alternatives excludes the choice of any of the others, the alternatives are called mutually exclusive. Typically, the alternatives being considered require the investment of different amounts of capital, and their annual revenues and costs may vary. Because different levels of investment normally produce varying economic outcomes, we must perform an analysis to determine which one of the mutually exclusive alternatives is preferred and, consequently how much capital should be invested. Five of the basic methods discussed in chapter 4 for analyzing cash flows are used in the analyses for comparing alternatives (PW, FW, AW, IRR and ERR method). The problem of deciding which mutually exclusive alternatives should be selected is made easier if we adopt a rule in which:

"The alternative that requires the minimum investment of capital and produces satisfactory functional results will be chosen unless the incremental (additional) capital associated with an alternative having a larger investment can be justified with respect to its incremental benefits".

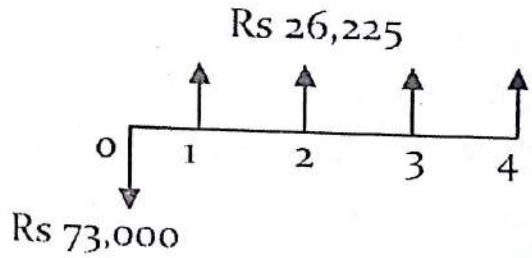
Under this rule, we consider the acceptable alternative that requires the least investment of the capital to be the base alternative. The investment of additional capital over that required by the base alternative usually results in increased capacity, increased quality, increased revenues, decreased operating expenses, or increased life. Therefore before additional money is invested, it must be shown that each avoidable increment of capital of capital can pay its own way relative to other available investment opportunities. Let us consider the two alternatives (projects):

	Alternatives	
	A	B
Capital Investment (Rs)	-60,000	-73,000
Annual Revenues (Rs)	22,000	26,225
Life (years)	4	4
MARR	10%	

Alternative A



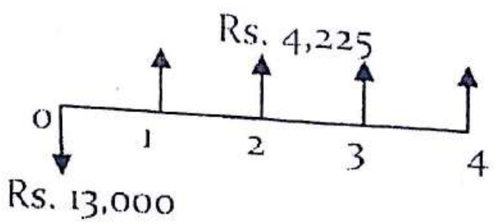
Alternative B



$(PW)_A$ = -60,000 + 22,000 (P/A, 10%, 4) = Rs. 9738

$(PW)_B$ = -73,000 + 26,225 (A/P, 10%, 4) = Rs. 10,131

Since PW_A >0, it is the base alternative and would be selected unless additional (incremental) capital associated with Alternative B is justified. In this case, Alternative B is preferred to A because it has a greater PW value.



(Alternative B minus Alternative A)

Hence the extra benefit obtained by investing the additional Rs. 13,000 of capital in B have a present worth of Rs. 10,131 – Rs. 9738 = Rs. 393.

$(PW)_{diff} = -13,000 + 4225(P/A, 10\%, 4) = Rs.\ 393.$

i.e. the Alternative B is in surplus of $ 393 at time zero.

"A firm always tries to increase investment by maximizing PW if justified"

Analysis Period

The analysis period is a time span over which the economic effects of an investment will be evaluated. The analysis period may also be called the *study period* or *planning horizon*. The length of the analysis period is determined by company policy, the service period, the useful life of the shorter lived alternative, the useful life of longer lived alternative etc.

Useful Life

The useful life of an asset is the time period during which it is kept in productive use in a trade or business.

The useful lives of alternatives being compared, relative to the selected study period, can involve two situations.

Case 1: Useful lives are the same for all alternatives and equal to the study period.

Case2: Useful lives are different among the alternatives and at least one does not matches the study period.

6.2 Case 1: Useful lives are the same for all alternatives and equal to the study period.

When the useful lives of alternative are equal to the selected study period, adjustments to the cash flow are not required. In this case, we compute the equivalent worth for each project and select the one with the highest worth.

Equivalent worth Method

When equivalent worth methods are used, consistency of alternative selection results from this equivalency relationship. Also the economic ranking of mutually exclusive alternatives will be the same using the three methods. Consider the general case of two alternatives, A and B. If

PW_A (i %) < PW_B (i %)

PW_A (i %) {A/P, i%, N}< PW_B (i %) {A/P, i%, N}

AW_A (i %) < AW_B (i %)

Similarly,

PW_A (i %) {F/P, i%, N) < PW_B (i %) {F/P, i%, N)

FW_A (i %) < FW_B (i %)

Ranking is always consistent in the equivalent method.

Example 6.1

Consider the mutually exclusive alternatives

	Option 1	Option 2	Option 3
Investment cost (Rs)	2,69,000	3.19,000	3.30,000
Annual net savings (Rs)	81,500	88,500	98,300
Useful life (years)	5	5	5
Salvage value (Rs.)	100,000	120,000	120,000

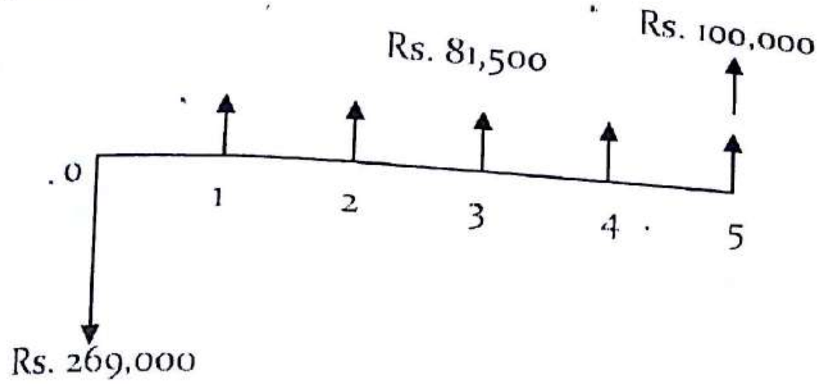
Which option would be selected based on equivalent worth method at i=12%

Solution

Since useful life of each alternative is 5 years, simply calculate the equivalent worth of each option.

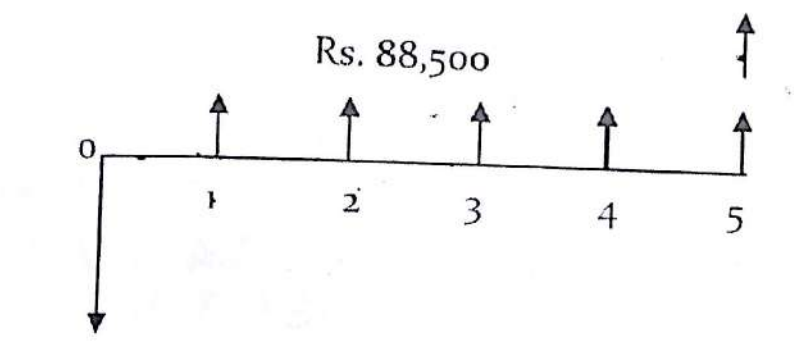
Using PW formulation

Option 1



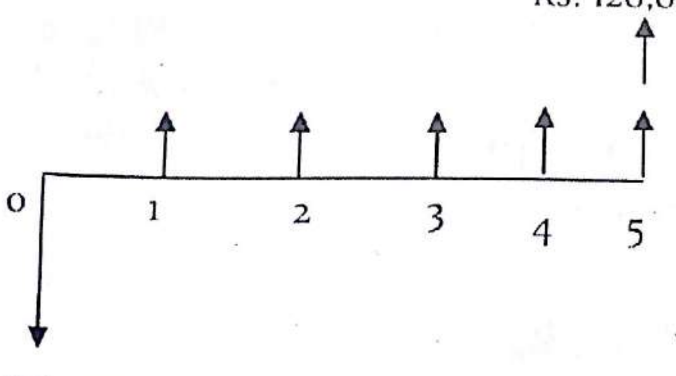
$PW_1 = -2,69,000 + 81,500\ (P/A, 12\%, 5) + 100,000\ (P/F, 12\%, 5)$
$= \textbf{Rs.81,532.}$

Option 2



$PW_2 = -3,19,800 + 88,500\ (P/A, 12\%, 5) + 120,000\ (P/F, 12\%, 5)$
$= \text{Rs. } 67,314$

Option 3



$PW_3 = -3,30,000 + 98,300\ (P/A, 12\%, 5) + 1,20,000\ (P/F, 12\%, 5)$
$= \text{Rs. } 91,640$

Here, $PW_3 > PW_1 > PW_2$

Option 3 is the most economical project.

Using FW formulation

FW_1 = -2, 69,000(F/P, 12%, 5) +81,500(F/A, 12%, 5) + 1,0 0,000
= - 4, 74,070 + 5, 17,757+ 1, 00,000
= **Rs. 1, 43,687**

FW_2 = -3, 19,000(F/P, 12%, 5) +88,500(F/A, 12%, 5) + 1, 0 0,000
= - 5, 62,187 + 5, 62,227+ 1, 20,000
= **Rs.1, 20, 040**

FW_3 = -3, 30,000(F/P, 12%, 5) +98,300(F/A, 12%, 5) + 1, 20,000
= - 5, 81,559 + 6, 24,480+ 1, 20,000
= **Rs. 1, 62,921**

Here, $FW_3 > FW_1 > FW_2$

Option 3 is most economical

Using AW formulation

AW_1 = -2, 69,000(A/P, 12%, 5) +81,500 + 1, 00,000(A/F, 12%, 5)
= 74, 620 + 81,500 + 15, 740 = **Rs. 22,620**

AW_2 = -3, 19,000 (A/P, 12%, 5) +88,500 + 1, 20,000(A/F, 12%, 5)
= -88,490 + 88,500 + 18,888 = **Rs. 18,898**

AW_3 = -3, 30, 000 (A/P, 12%, 5) + 98,300 + 1, 20,000(A/F, 12%, 5)
= -91,542 + 98,300 + 18,888 = **Rs. 25,646**

Here, $AW_3 > AW_1 > AW_2$

Option 3 is most economical

Rate of Return Method

Under Equivalent worth Method, the mutually exclusive project with the highest worth figure was preferred. Unfortunately, the same principle cannot be applied to IRR, ERR and BCR analysis. The project with highest IRR, ERR and BCR may not be the preferred alternative. Let us consider the two mutually exclusive alternatives with 1 year of service life.

N	A_1	A_2
0	-Rs1,000	-Rs5,000
1	Rs 2,000	Rs 7,000
IRR	100%	40%
BCR (10%)	1.82	1.27
PW (10%)	Rs 818	Rs 1364

We can see that, A2 is preferred over A1 by PW method and A1 is preferred over A2 by IRR and BCR methods.

This inconsistency is because equivalent worth method (AW, PW, FW) are absolute method where as IRR and ERR is relative (percentage) measure and cannot be applied in the same way. For this purpose *Incremental Analysis* should be done.

Incremental Analysis

Incremental Analysis evaluates difference, or the 'increment" between two or more mutually exclusive alternatives. As we have learned, the PW and AW techniques can be used to do incremental analysis. When independent projects are evaluated, no incremental analysis is necessary between projects. Each project is evaluated separately from others, and more than one can be selected.

Steps for Incremental Analysis

1. **Identify** all the alternatives
2. **Compute the IRR/ERR/BCR** of each alternative. Any alternative with IRR<MARR/ERR<MARR/BCR<1 should be rejected.
3. **Order alternatives in increasing order** of investment cost to ensure that the increments have cash flow corresponding to investments.
4. **Establish a base alternative:** Alternative having least capital investment is established as the base

alternative and should have been pre qualified i.e. IRR> MARR/ERR>MARR/BCR>1

5. **Perform an incremental analysis** between the base alternative and the alternative with the next higher initial cost. If the incremental IRR ≥ MARR/ ERR ≥ MARR/ BCR ≥ 1 "reject" the base alternative and "accept" the higher cost alternative and "retain" it as base alternative.
6. **Select the next higher cost alternative** and perform the incremental analysis until all the alternatives have been evaluated.

"Evaluation should always be done based on same study period for all alternatives"

Decision Rule

IF, IRR_{B-A}> MARR/ ERR_{B-A}> MARR/ BCR_{B-A}> 1, Select B (higher first cost alternative)

IRR_{B-A}= MARR/ ERR_{B-A}= MARR/ BCR_{B-A}= 1, Select either one

IRR_{B-A}< MARR/ ERR_{B-A}<MARR/ BCR_{B-A}< 1, Select A (lower first cost alternative)

For above example,

Year	A_2 -A_1	Remark
0	-Rs 4,000	
1	Rs 5,000	
IRR	25%>MARR	Select A_2
BCR	1.14 >1	Select A_2

Example 6.2

The cash flows for the two mutually exclusive alternatives are given as follows:

EOY	A	B
0	- Rs. 3,000	- Rs. 12,000

1	Rs. 1,350	Rs. 4,200
2	Rs. 1,800	Rs. 6,225
3	Rs. 1,500	Rs. 6,330

Which project would be selected based on IRR criterion, at MARR = 10%

Solution

Calculating the IRR of each alternative.

IRR_A = 25% > 10% (MARR) **(Justified)**

IRR_B = 17.43% > 10% (MARR) **(Justified)**

EOY	A	B	B – A
0	- Rs. 3,000	- Rs. 12,000	- Rs. 9,000
1	Rs. 1,350	Rs. 4,200	Rs. 2,850
2	Rs. 1,800	Rs. 6,225	Rs. 4,425
3	Rs. 1,500	Rs. 6,330	Rs. 4,830
IRR	25%	17.43%	?

Performing the incremental analysis.

Let us consider the mutually exclusive alternative as in the table above.

Why did we choose to look at the increment B-A instead of A-B?

We want the first flow of the incremental cash flow series to be negative (investment flow) so that we can calculate IRR. By subtracting the lower initial investment project from the higher, we guarantee that the first increment will be an investment flow. If we ignore the investment ranking, we might end up with and increment that involves borrowing cash flow and has no internal rate of return.

If we choose A –B, we have to borrow Rs. 9000, the interest rate of A-B= 15%. It means that we are loosing or paying 15% interest rate by not investing the Rs. 9000 in B. If we invest in

A We are saving Rs. 9000 now, but we are giving up the opportunity of making Rs. 2850 at the end of 1st year, Rs. 4428 at 2nd year and Rs. 4830 at the 3rd year.

Calculating IRR of the incremental Cash flow.

$PW(i^*)_{B-A} = 0$

$-9000 + 2850(P/F, i^*, 1) + 4425(P/F, i^*, 2) + 4830(P/F, i^*, 3) = 0$

$(i^*)_{B-A} = 15\% > 10\%$ (MARR)

Hence, Option B is selected.

Example 6.3

Consider the following three set of mutually exclusive alternatives:

Alternatives			
EOY	D1	D2	D3
0	- Rs. 2000	- Rs. 1000	- Rs. 3000
1	Rs. 1500	Rs. 800	Rs. 1500
2	Rs. 1000	Rs. 500	Rs. 2000
3	Rs. 800	Rs. 500	Rs. 1000

Which project would you select based on IRR, ERR and BCR methods on incremental investment assuming that MARR = ε=15%?

Solution

Calculate the IRR of the each project.

IRR of D_1

$PW = 0$

$-2{,}000 + 1{,}500(P/F, i^*, 1) + 1{,}000(P/F, i^*, 2) + 800(P/F, i^*, 3) = 0$

i^* = 34.37% >15% (MARR) (Justified)

IRR of D_2

$PW = 0$

$-1{,}000 + 800(P/F, i^*, 1) + 500(P/F, i^*, 2) + 500(P/F, i^*, 3) = 0$

i^* = 40.76% >15% (MARR) (Justified)

IRR of D_3

$PW = 0$

-3,000 +1,500 (P/F, i*, 1) + 2,000 (P/F, i*, 2) + 1,000 (P/F, i*, 3) = 0

i* = 24.81% >15% (MARR) (Justified)

Calculate the BCR of the each project using PW formulation

BCR of D_1 = PW (Benefits)/ PW (Costs)

PW (Benefits) = 1,500 (P/F, 15%, 1) + 1,000 (P/F. 15%, 2) + 800 (P/F. 15%, 3)

= 1,500*0.8696 + 1,000 *0.7561 + 800 *0.6575

= Rs. 2,586.5

PW (Costs) = Rs. 2,000

BCR of D_1 = 2,586.5 /2,000 = **1.293 > 1 (Justified)**

BCR of D_2 = PW (Benefits)/ PW (Costs)

PW (Benefits) = 800 (P/F, 15%, 1) + 500 (P/F, 15%, 2) + 500 (P/F, 15%, 3)

= 800*0.8696 + 500 *0.7561 + 500 *0.6575

= Rs. 1,402.48

PW (Costs) = Rs. 1,000

BCR of D_2 = 1,402.48 /1,000 = **1.402 > 1 (Justified)**

BCR of D_3 = PW (Benefits)/ PW (Costs)

PW (Benefits) = 1,500 (P/F, 15%, 1) + 2,000 (P/F, 15%, 2) + 1,000 (P/F, 15%, 3)

= 1,500*0.8696 + 2000 *0.7561 + 1000 *0.6575 = Rs. 3,474.1

PW (Costs) = Rs. 3,000

BCR of D_3 = 3,474.1 /3,000 = **1.15 > 1 (Justified)**

Calculate the ERR of the each project using PW formulation

ERR of D_1

- Compound all cash inflow to the future value
  3000 (F/P, 15%, 2) + 1500 (F/P, 15%, 1) + 800 = Rs. 3933.75
- Discount all cash outflow to the present value = 2000
- Making the equivalence of two equation
  $2000 (1+i')^3 = 3933.75$
  $(1+i)^3 = 1.966$
  i = 25.29% >MARR (acceptable)

ERR of D_2

- Compound all cash inflow to the future value
  800 (F/P, 15%, 2) + 500 (F/P, 15%, 1) + 500 = Rs. 2133
- Discount all cash outflow to the present value = 1000
- Making the equivalence of two equation
  $1000 (1+i')^3 = 2133$
  $(1+i)^3 = 2.133$
  i = 28.72% >MARR (acceptable)

ERR of D_3

- Compound all cash inflow to the future value
  1500 (F/P, 15%, 2) + 2000 (F/P, 15%, 1) + 1000 = Rs. 5283.75
- Discount all cash outflow to the present value = 3000
- Making the equivalence of two equation
  $3000 (1+i')^3 = 5283.75$
  $(1+i)^3 = 1.76$
  i = 20.76% >MARR (acceptable)

Performing the Incremental Analysis

Incremental IRR

- Select D_2 as the base alternative because it has the lower initial cost (Rs. 1000).

- Compare D_1 with base alternative D_2 which is the next higher initial cost (Rs. 2000).
- Calculate the incremental cost and incremental benefits (D_1-D_2)
- Calculate the **IRR** of the incremental cash flow.

  -1,000 +700 (P/F, i*, 1) +500 (P/F, i*, 2) + 300 (P/F, i*, 3) =0

  i*= 27.61 >15% (MARR)
- Eliminate D_2 from the consideration because alternative D_1 gives the higher return.
- Select D_1 as the base alternative which initial cost is lower than the next remaining alternative D_3.
- Compare alternative D_1 with D_3 and compute incremental cash flow. (D_3-D_1)
- Calculate the IRR of the D_3-D_1.

EOY	D2	D1-D2	D3-D1
0	-1000	-1000	-1000
1	800	700	0
2	500	500	1000
3	500	300	200
Incremental IRR	40. 76%	27.61%	8.8%
Is increment justified		Yes	No
		Select D1 Reject D2	Reject D3 Select D1

Select D_1 as the best alternative

Incremental BCR

Calculate the **BCR** of the incremental cash flow

PW (benefits) = 700 (P/F, 15%, 1) + 500 (P/F, 15%, 2) + 300.(P/F, 15%, 3)

= 608.7 + 378.07 + 197.25 =1184.02

BCR = 1,184.02/1,000 = 1.18 >1Here D_2 is being eliminated from the consideration and D_1 is taken as the base alternative and compared with alternative D_3

EOY	D2	D1-D2	D3-D1
0	-1000	-1000	-1000
1	800	700	0
2	500	500	1000
3	500	300	200
Incremental BCR	1.402	1.18	0.88
Is increment justified		Yes	No
		Select D1 Reject D2	Reject D3 Select D1

Select D_1 as the best alternative

Incremental ERR

Calculate the ERR of the incremental cash flow.

{700 (F/P, 15%*, 2) +500 (F/P, 15%*, 1) + 300}=

1,000$(1+i')^3$

$(1+i')^3$ = 1.80075

i'=21.66% >MARR (acceptable)

Here, D1 is being eliminated from the consideration and again D2 is taken as base alternative and compared with alternative D3

EOY	D2	D1-D2	D3-D1
0	-1000	-1000	-1000
1	800	700	0
2	500	500	1000
3	500	300	200
Incremental ERR	28.72%	21.66%	9.13%
Is increment justified		Yes	No
		Select D1 ,Reject D2	Reject D3,Select D1

Select D1 as the best alternative

Example 6.3

An engineering firm is considering the following mutually exclusive alternatives

EOY	PROJECTS			
	A1	A2	A3	A4
0	- Rs. 2500	- Rs. 1200	- Rs. 3600	- Rs. 2000
1	Rs. 1200	Rs. 400	Rs. 1700	Rs. 800
2	Rs. 1400	Rs. 800	Rs. 2000	Rs. 700
3	Rs. 1500	Rs. 1000	Rs. 1600	Rs. 850

Which project would you select based on IRR and BCR methods. Assuming MARR = 20% per year.

Solution

Step 1: **Calculating the IRR of the each project**

Alternative A_1

PW (i*) = -2500 + 1200 (P/F, i*, 1) +2000 (P/F, i*, 2) +1600 (P/F, i*, 3)

By trial and error,

i* = 28.19%

Alternative A_2

PW (i*) = -2000 + 400 (P/F, i*, 1) + 800 (P/F, i*, 2) + 1000 (P/F, i*, 3)

By trial and error, i* = 31.84%

Alternative A_3

PW (i*) = -3600 + 1700 (P/F, i*, 1) + 1400 (P/F, i*, 2) + 1500 (P/F, i*, 3)

By trial and error, i* = 22.33%

Alternative A_4

PW (i*) = -2000 + 800 (P/F, i*, 1) + 700 (P/F, i*, 2) + 850 (P/F, i*, 3)

By trial and error, i* = 8.43%

Step 2: **Comparing with MARR=20%**

- Alternative A_1, $i^* = 28.19 >$ MARR (20%), **accepted**
  Alternative A_2, $i^* = 31.84 >$ MARR (20%), **accepted**
  Alternative A_3, $i^* = 22.33 >$ MARR (20%), **accepted**
  Alternative A_4, $i^* = 8.43 <$ MARR (20%), **rejected**
  We are now left with three projects i.e. A_1, A_2 and A_3

Calculating the BCR of the each project

BCR_{A1}= PW (Benefits) / PW (Costs)

PW (Benefits) = 1200(P/F, 20%, 1) + 1400 (P/F, 20%, 2) + 1500 (P/F, 20%, 3)

= 3314.81

BCR_{A1}= 3314.81 / 2500 = **1.325 >1 (justified)**

BCR_{A2}= PW (Benefits) / PW (Costs)

PW (Benefits) = 400(P/F, 20%, 1) + 800 (P/F, 20%, 2) + 1000 (P/F, 20%, 3)

= 1467.59

BCR_{A2}= 1467.59/ 1200 = **1.22 >1 (justified)**

BCR_{A3}= PW (Benefits) / PW (Costs)

PW (Benefits) = 1700(P/F, 20%, 1) + 2000 (P/F, 20%, 2) + 1600 (P/F, 20%, 3)

= 3731.48

BCR_{A3}= 3731.48/ 3600 = **1.036 > 1 (justified)**

BCR_{A4}= PW (Benefits) / PW (Costs)

PW (Benefits) = 800(P/F, 20%, 1) + 700 (P/F, 20%, 2) + 850 (P/F, 20%, 3)

= 1644.67

BCR_{A4}= 1644.67/ 2000 = **0.822 < 1 (not justified)**

Step 3: **Performing the incremental analysis**

Taking Alternative A_2 as the base alternative (alternative having lowest investment)

Comparative Analysis of Alternatives

EOY	A2	A1-A2	A3-A1
0	- Rs. 1200	- Rs. 1300	- Rs. 1100
1	Rs. 400	Rs. 800	Rs. 500
2	Rs. 800	Rs. 600	Rs. 600
3	Rs. 1000	Rs. 500	Rs. 100
Incremental IRR	31.84%	23.87%	5.39%
Incremental BCR	1.22	1.05	0.75
Is increment justified	Yes	Yes	No

Decision

- When A_2 is compared with A_1, IRR is 23.87% > MARR (20%) and BCR is 1.05 > 1, we select alternative A_1 leaving behind A_2.
- Now the base alternative is A_1 and compared with A_3 which gives IRR = 5.39% < MARR (20%) and BCR < 1, we select Alternative A_1 leaving behind A_3.

Hence, **Select project A_1 from IRR method as well as B/C ratio method**

6.3 Case 2: Analysis period differs from the project live and useful lives are unequal among the alternatives.

In previous section, we assumed the simplest scenario possible when analyzing mutually exclusive projects. The project had useful lives equal to each other and to the required service period. In practice this is seldom the case. Often project lives do not match the required analysis period and/or do not match each other. For example, two machines may perform exactly the same function, but one lasts longer than the other, and both of them last longer than the analysis period for which they are being considered. In following sections and examples, we will develop some techniques for dealing with these complications.

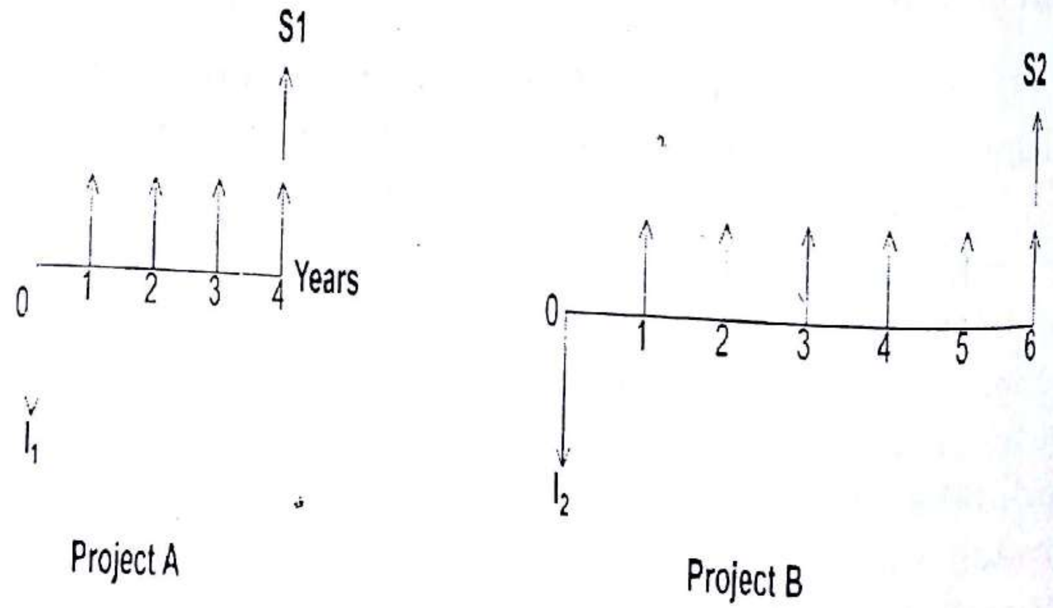
The two types of assumptions *repeatability assumption* and *co-terminated assumption* are used for the economic comparison of mutually exclusive alternatives.

Repeatability Assumption

- Two alternatives having different useful life are *changed into projects having same useful* life by expanding their life up to least common year.
- The *study period* is equal to the least common multiple (LCM) of the lives of alternatives.
- The economic consequences that are estimated to happen in an alternative's initial life span will also happen in all succeeding life spans.

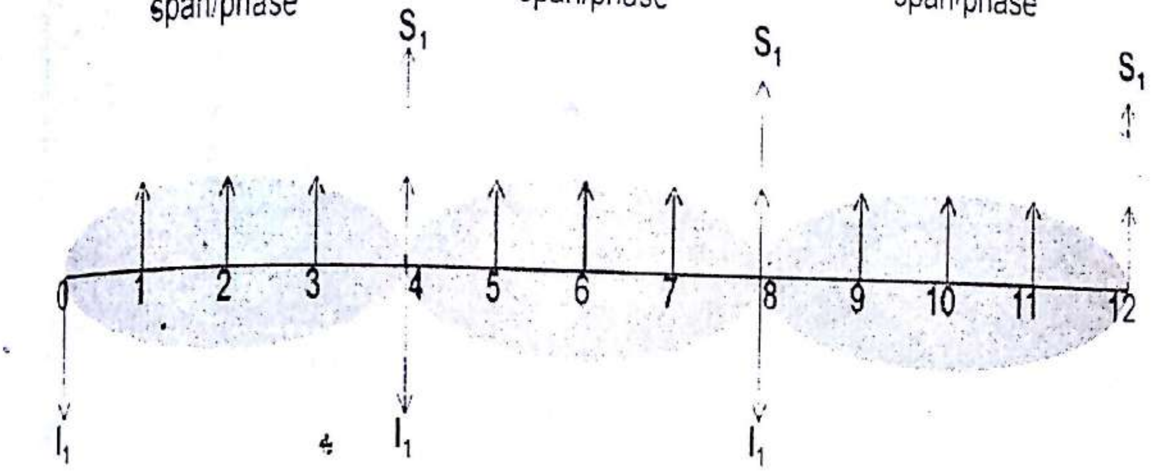
Let us consider the two projects having the following cash flow.

Lets consider the following project



LCM of 4 and 6 is 12 years, so we assume study period as 12 years

Project A is repeated 3 times



Project B is repeated 2 times

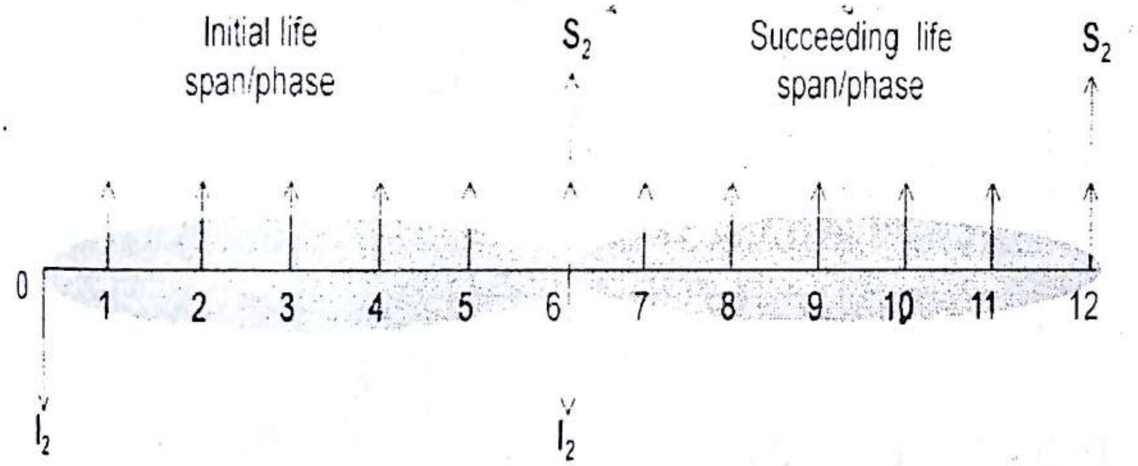


Fig 6.1: Cash flow diagram for repeatability assumption

Example 6.4

A project engineer with Environment Care is assigned to start up a new office in a city. Two lease options are available, each with a first cost, annual lease cost, and deposit-return estimates shown below. Determine which lease option should be selected on the basis of a present worth, future worth and annual worth comparison, if the MARR is 15% per year.

	Location A	Location B
First Cost (Rs)	-15,000	-18,000
AOC (Rs)	-3,500	-3,100
Deposit	1,000	2,000
Return(Rs)		
Life (Years)	6	9

Solution

Here useful life of location A and location B is 6 and 9 respectively.

Study period = LCM of 6 and 9 i.e. 18 years

Location A is repeated for 3 times and Location B is repeated for 2 times

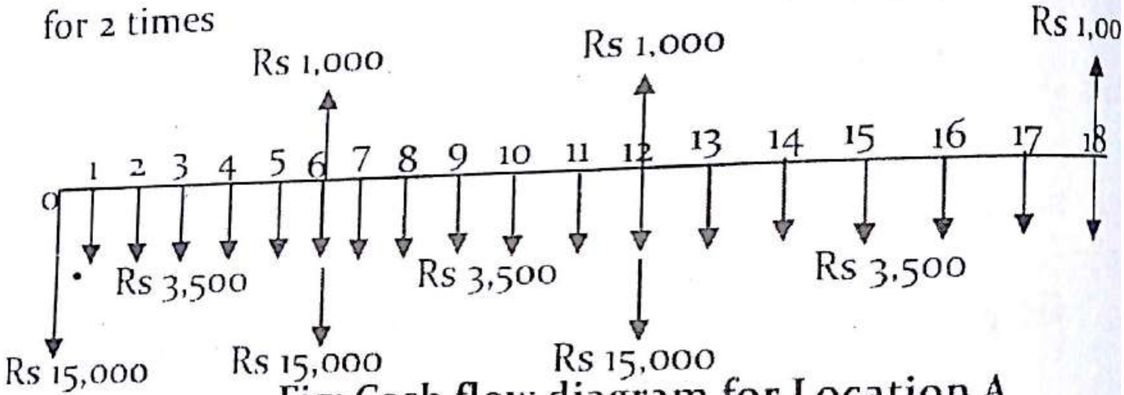


Fig: Cash flow diagram for Location A

Using PW formulation

$PW_A(15\%) = -15{,}000\{1+ (P/F, 15\%, 6) + (P/F, 15\%, 12)\} +1{,}000 \{(P/F, 15\%, 6) + (P/F, 15\%, 12) + (P/F, 15\%, 18)\} - 3{,}500(P/A, 15\%, 18)$

$= - 15{,}000 \{1+ (1+0.15)^{-6} + (1+0.15)^{-12}\} +1{,}000 \{(1.15)^{-6} + (1.15)^{-12} + (1.15)^{-18}\} - 3{,}500 ((1+0.15)^{18}-1)/ (1+0.15)^{18} *0.15)$

$\mathbf{PW_A(15\%) = - Rs.45,036}$

Using FW formulation

$FW_A(15\%) = -15{,}000\{(F/P, 15\%, 18) + (F/P, 15\%, 12) + F/P, 15\%, 6)\} +1{,}000 \{(F/P, 15\%, 6) + (F/P, 15\%, 12) +1\} - 3{,}500(F/A, 15\%, 18)$

$= -15{,}000\ \{(1+0.15)^{18}) + (1+0.15)^{12} + (1+0.15)^{6}\} + 1{,}000\ \{(1.15)^{6} + (1.15)^{12} + 1\} - 3{,}500\ ((1+0.15)^{18}-1)/\ 0.15)$

FW_A (15%) = - Rs.5, 57,345. 40

Using AW formulation

$AW_A (15\%) = PW_A (15\%)\ \{A/P, 15\%, 18)$

$= 45{,}036 * 0.1632$

AW_A (15%) = - **Rs. 7,349.87**

OR

AW of alternative can be found out by directly calculating the AW of initial (given) cash flow of useful life.

$AW_A (15\%) = -15{,}000\ (A/P, 15\%, 6) - 3{,}500 + 1{,}000\ (A/F, 15\%, 6)$

$= -15{,}000 * 0.2642 - 3{,}500 + 1{,}000 * 0.1142$

AW_A (15%) = **Rs. 7,349.80**

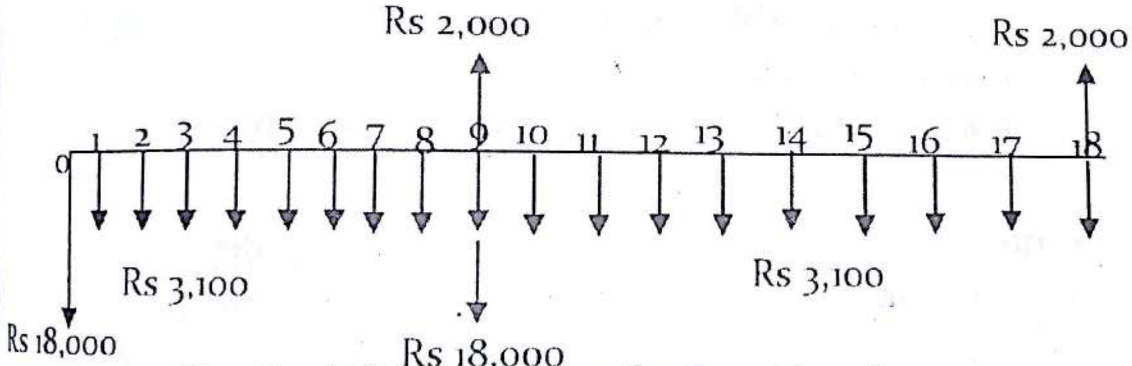


Fig: Cash flow diagram for Location B

Using PW formulation

$PW_B (15\%) = -18{,}000 - 18{,}000(P/F, 15\%, 9) + 2000\ (P/F, 15\%, 9) + 2000\ (P/F, 15\%, 18) - 3{,}100(P/A, 15\%, 18)$

$= -18{,}000\ \{1 + (1+0.15)^{-9}\} + 2{,}000\ \{(1.15)^{-9} + (1.15)^{-18}\} - 3{,}100\ ((1+0.15)^{18}-1)/\ (1+0.15)^{18} * 0.15)$

PW_B (15%) = - **Rs.41, 384**

Using FW formulation

$FW_B (15\%) = -18{,}000\{(F/P, 15\%, 18) + (F/P, 15\%, 9)\} + 2{,}000\ \{(F/P, 15\%, 9) + 1\} - 3{,}100(F/A, 15\%, 18)$

$= -18{,}000\ \{(1+0.15)^{18}) + (1+0.15)^{9}\} + 2{,}000\ \{(1.15)^{9} + 1\} - 3{,}500\ ((1+0.15)^{18}-1)/\ 0.15)$

$FW_B(15\%)$ = - Rs. 5, 12,138.24

Using AW formulation

$AW_B(15\%) = PW_B(15\%)\ \{A/P, 15\%, 18)$

$= 41,384 * 0.1632$

$AW_B(15\%)$ = - Rs. 6,753.86

OR

AW of alternative can be found out by directly calculating the AW of initial (given) cash flow of useful life.

$AW_B(15\%) = -18,000\ (A/P, 15\%, 9) - 3,100 + 2,000\ (A/F, 15\%, 9)$

$= -18,000 * 0.2096 - 3,100 + 2,000 * 0.0596$

$AW_B(15\%)$ = - Rs. 6,753.6

Here, $PW_B(15\%) > PW_A(15\%)$

$FW_B(15\%) > FW_A(15\%)$

$AW_B(15\%) > AW_A(15\%)$, **Select Location B (Ans)**

Example 6.4

The following data have been estimated for two mutually exclusive investment alternatives A and B, associated with a small engineering project for which revenues as well as expenses are involved. They have useful lives of four and six years respectively. If the MARR=10%per year, show which alternative is more desirable by using present worth and annual worth method. Use repeatability assumption.

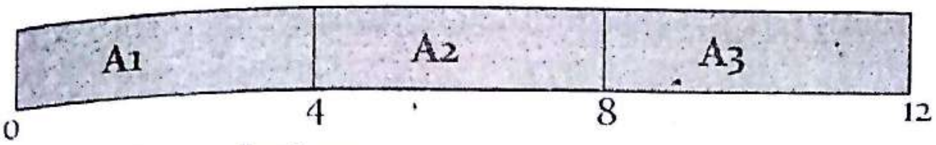
	A	B
Capital Investment (Rs)	3,500	5,000
Annual revenue (Rs)	1,900	2,500
Annual expenses (Rs)	645	1,020
Useful Life (Years)	4	6

Solution

Here useful life of Alternative A and Alternative B is 4 and 6 respectively.

Study period = LCM of 4 and 6 i.e. 12 years

Three cycles of Alternative A

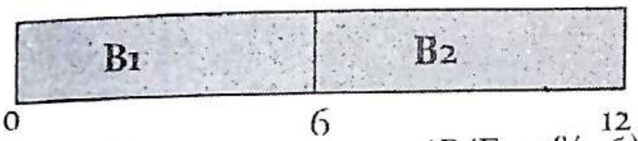


Using PW formulation

$PW(10\%)_A = -3{,}500 - 3{,}500\,[(P/F, 10\%, 4) + (P/F, 10\%, 8)] + (1900 - 645)\,(P/A, 10\%, 12)$

$PW(10\%)_A$ = **Rs 1,028**

Two cycles of alternative B



$PW(10\%)_B = -5{,}000 - 5{,}000\,(P/F, 10\%, 6) + (2{,}500 - 1{,}020)\,(P/A, 10\%, 12)$

$PW(10\%)_B$= **Rs 2,262**

Based on the PW method we should **select Alternative B** because it has the largest value (Rs.2, 262)

Using AW method

$AW(10\%)_A = -3{,}500\,(A/P, 10\%, 4) + (1{,}900 - 645) =$ **Rs. 150.75**

$AW(10\%)_B = -5{,}000\,(A/P, 10\%, 6) + (2{,}500 - 1{,}020) =$ **Rs. 332**

Based on the AW method we should **select Alternative B** because it has the largest value (Rs.332)

Co-terminated Assumption

The co-terminated uses a finite and identical study period for all alternatives. This planning horizon combined with appropriate adjustments to the estimated cash flows plus the alternatives on a common and comparable basis. The planning horizon chosen could be

- Life of shorter lived alternative.

- Life of longer lived alternative.
- Less than the shorter lived alternatives.
- Greater than the longer lived alternative.
- In between the shortest and longest lived alternatives.

Two cases are involved in the co-terminated assumption

Case 1: Study period longer than the Useful life.

Case 2: Study period shorter than the Useful life.

Case 1: Study period longer than the Useful life.

Project lives rarely conveniently coincide with a firm's predetermined required analysis period; they are often too long or too short. The case of project lives that are too long is the easier one to address. A common instance of project lives that are longer than the analysis period occurs in the construction industry, where a building project may have a relatively short completion time, but the equipment purchased has a much longer useful life.

Two assumptions are considered:

1. Cash flow accumulated at the end of the useful life will be reinvested for the extended periods.
2. Replacement/Reinvestment is necessary for remaining period (study period –useful life) and economic consequences that are estimated to happen in an alternative's initial life span will also happen in all succeeding life spans (As in repeatability assumption)

Example 6.5

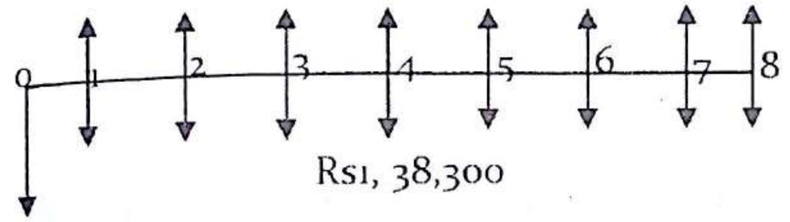
Consider the following mutually exclusive projects. MARR =10%

	A	B
Investment	Rs 350000	Rs 500000
Annual revenue	Rs 190000	Rs 250000
Annual cost	64500	138300
Useful life	4 years	8 years
Salvage value	0	0

Which alternative is more desirable based on the co-terminated assumption.

Solution

Taking analysis period as 8 years (the value should be taken in such a way that the study period is either equal to or greater than useful lives of all the alternatives). If lesser useful life is taken than we have to curtail down the cash flow to the end of study period and suitable market value should be assigned to the alternatives.



Cash flow diagram of Project B

There is no adjustment required for alternative B. The adjustment is required in case of A, which study period is 4 years greater than its useful life.

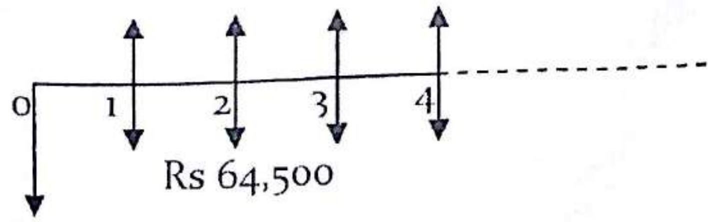
Considering assumption 1

FW_B (10%) = -5, 00, 000 (F/P, 10%, 8) + (25,000- 1, 38,300) (F/A, 10%, 8)

$= -5,00,000\ \{(1.1)^8\} + 1,11,700\ \{(1.1)^8 - 1/0.1\}$

$= -10,71,794.405 + 12,77,388.701$

FW_B (10%) = **Rs. 2, 05,594.2958**



Cash flow diagram of Project A

$FW_A(10\%) = \{-3,50,000(F/P, 10\%, 4) + (1,90,000 - 64500)(F/A, 10\%, 4)\}(F/P, 10\%, (8-4))$

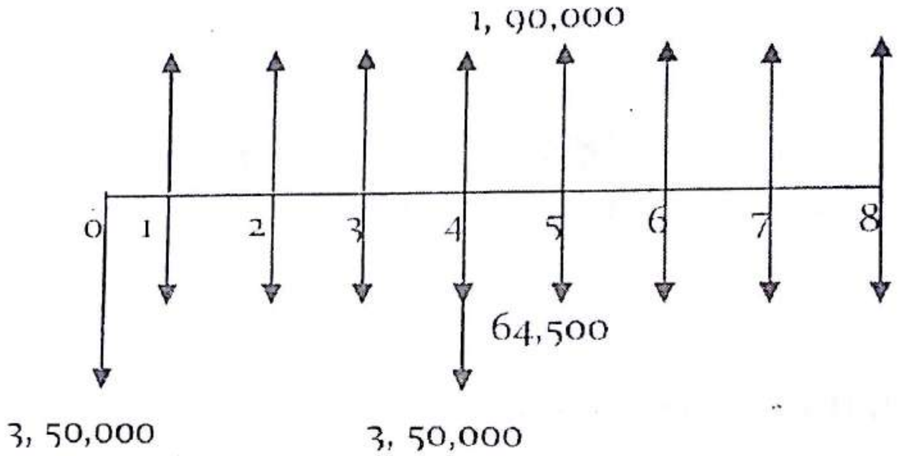
$= \{-3,50,000(1.1)^4 + 1,25,500((1.1)^4 - 1)/0.1\}(1.1)^4$

$= (-5,12,435 + 5,82,445.5)(1.1)^4$

$FW_A(10\%) = 1,02,502.37$

$FW_B(10\%) > FW_A(10\%)$, select alternative B

Considering assumption 2



Cash flow diagram of Project A

$FW_A(10\%) = -3,50,000(F/P, 10\%, 8) - 3,50,000(F/P, 10\%, 4) + (1,90,000 - 64,500)(F/A, 10\%, 8)$

$FW_A(10\%) = Rs.\ 1,72,510$

$FW_B(10\%) > FW_A(10\%)$, select alternative B

Case 2: Study period shorter than the Useful life.

When the project lives are shorter than the required service period, we must consider how, at the end of project lives, we will satisfy the rest of the required service period. The most common technique is to truncate the alternative at the end of the study period using an estimated market value. This assumes that the disposable assets will be sold at the end of the study period at that value.

The Imputed Market Value

Obtaining a current estimate from the market place for a piece of equipment or another type of asset is the preferred procedure in engineering practice when a market value at time T< useful life is required. This approach, however, may not be feasible in some cases. For example, a type of asset may have low turnover in the market place and information for recent transactions is not available. Hence, it is sometimes necessary to estimate the market value for an asset without current and representative historical data.

The imputed market value technique which is sometimes called the implied market value can be used for this purpose as well as for comparison with market place values when current data are available. If an imputed market value is needed for a piece of equipment, say at the end of T<useful life, the estimate is calculated based on the sum of two parts as follows.

MV_T = [PW at end of year T of remaining capital recovery amounts] + [PW at end of year T of original market value at end of useful life]

Example 6.6

Use the imputed market value at the end of year five if the useful life of the alternative is nine years, capital investment is Rs. 47,600,market value at the end of useful life is Rs. 5000, and MARR =20%.

Solution

PW (20%) $_{CR}$ = [47,600 (A/P, 20%, 9) – 5000 (A/F, 20%, 9)] * (P/A, 20%, 4)

= Rs. 29,949

Compute the PW at the end of year five, based on the original MV at the end of useful life (9 years)

PW (20%) $_{MV}$ = 5,000 (P/F, 20%, 4) = Rs. 2,412

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Then, the estimated market value at the end of year five ($T_{=5}$) is

MV_5 = PW (20%) $_{CR}$ + PW (20%) $_{MV}$

=29,949 + 2,412 = Rs. 32,361. (Ans)

Example 6.7

Using co-terminated assumption recommend the best project taking study period as 5 years (TU 2063)

Project	A	B
Initial Investment (Rs)	3,50,000	5,00,000
Annual Revenues (Rs)	1,30,000	1,75,000
Annual cost (Rs)	15,000	25,000
Salvage value (Rs)	35,000	50,000
Useful life	5 years	8 years
MARR	10%	

Solution

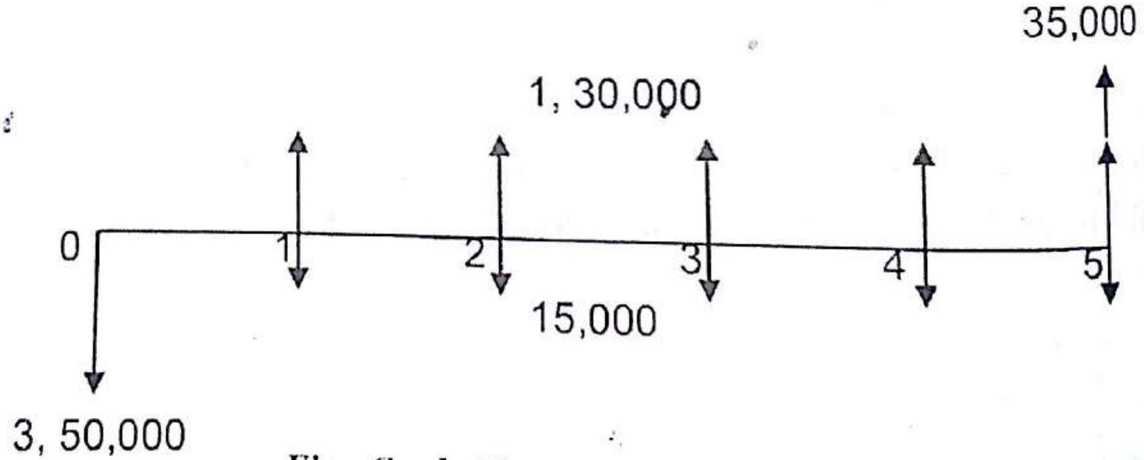


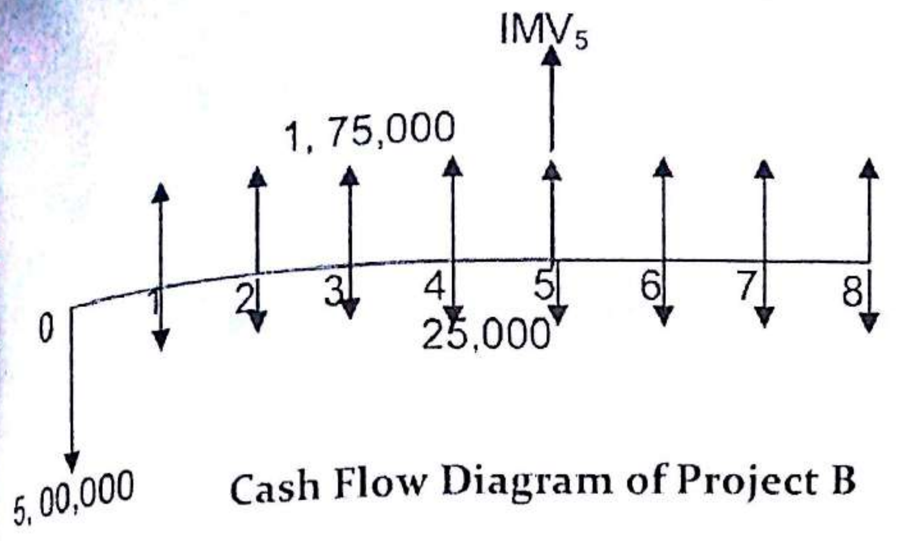
Fig: Cash Flow Diagram of Project A

Using FW formulation,

FW_A (10%) = -3, 50,000(F/P, 10%, 5) + (1, 30,000 -15,000) (F/A, 10%, 5) + 35,000

= - 5, 63,678.5 + 702086.5 + 35,000

= **Rs. 1, 73,408.**



Cash Flow Diagram of Project B

Applying imputed market value calculation,

CR (10%) = 5, 00,000 (A/P, 10%, 8) – 50,000 (A/F, 10%, 8)

= 93,722 – 4,372

= Rs. 89,350

Present worth (at year 5) of CR for remaining 3 years

PW_{CR} (10%) = 89,350 (P/A, 10%, 3)

= Rs. 2, 22,200

Present worth (at year 5) of Market value (MV) for remaining 3 years

PW_{MV} (10%) = 50,000(P/F, 10%, 3)

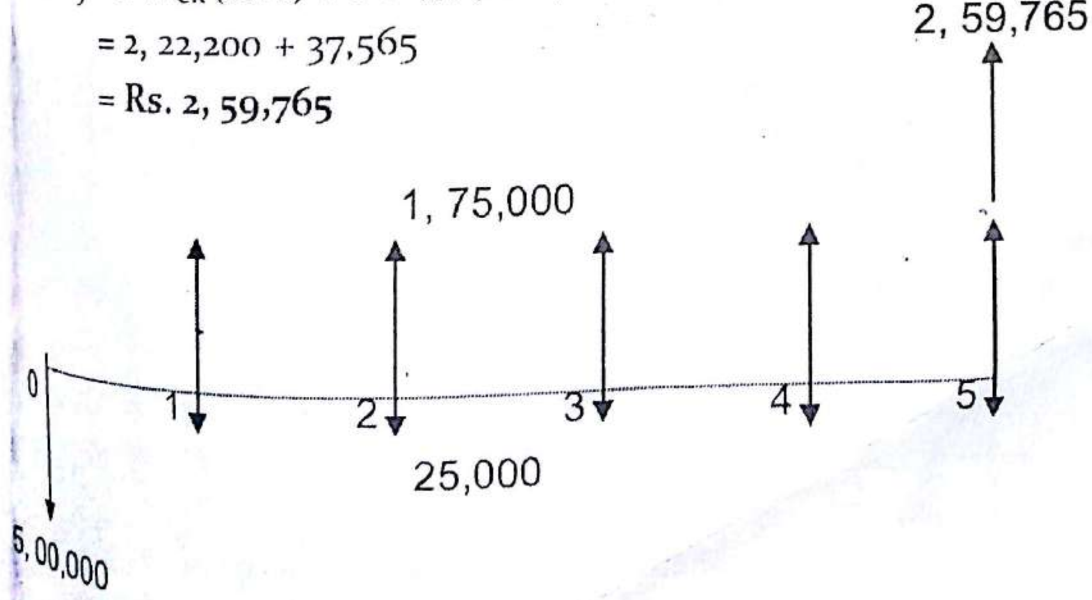
= Rs. 37,565

Market value at the study period. i.e. year 5

MV_5 = PW_{CR} (10%) + PW_{MV} (10%)

= 2, 22,200 + 37,565

= Rs. 2, 59,765



Revised Cash flow diagram of Alternative B

Using FW formulation,

$FW_B(10\%) = -5,00,000(F/P,10\%,5) + (1,75,000 - 25,000)$

$(F/A,10\%,5) + 2,59,765$

$= -8,05,255 + 9,15,765 + 2,59,765$

$= \text{Rs. } 3,70,275.$

FW_B (10%) $\geq$ FW_A (10%), Recommend Project B

IRR method for the Unequal project lives

The IRR method can also be used to compare projects with unequal lives, as long as we establish a common analysis period. This can be performed by using the Repeatability as well as co-terminated assumptions.

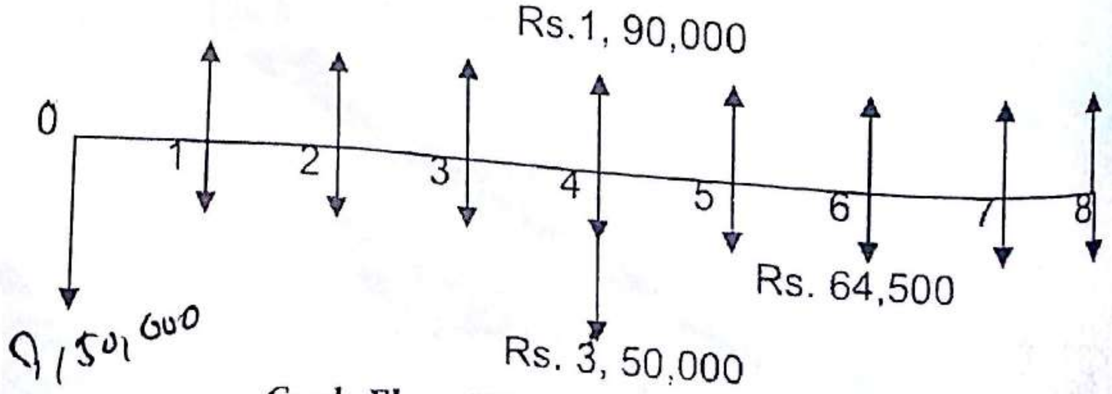
Example 6.8

Consider the following mutually exclusive investment projects A and B. Recommend the best project using IRR method.

Project	A	B
Investment (Rs)	3,50,000	5,00,000
Annual Revenues (Rs)	1,90,000	2,50,000
Annual Cost (Rs)	64,500	1,38,300
Useful life	4 yrs	8 yrs
MARR	10%	

Solution

Study Period = LCM of 4 and 8 = 8 years



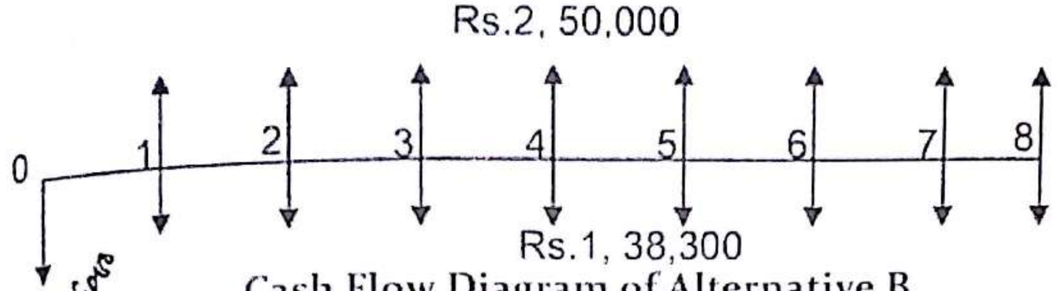
Cash Flow Diagram of Alternative A

$PW_A (i^* \%) = 0$

$-350{,}000 + (190{,}000 - 64500)\,(P/A, i^*\%, 8) - 350000\,(P/F, i^*\%, 4)$

$= 0$

$i^* = 16.2 >$ MARR (10%) (Accepted)



Cash Flow Diagram of Alternative B

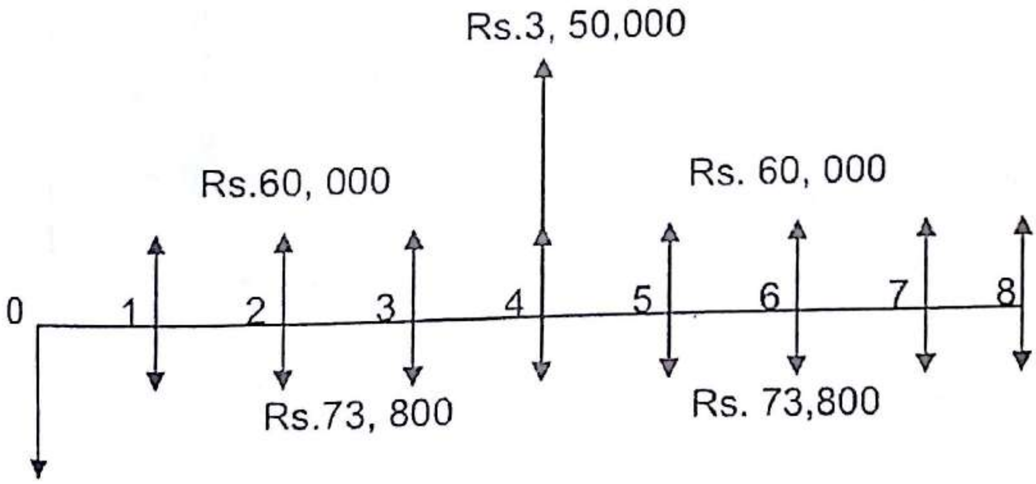
$PW_B (i^* \%) = 0$

$-500000 + (250{,}000 - 138300)\,(P/A, i^*\%, 8) = 0$

$i^* = 15.10 >$ MARR (10%) (Accepted)

Performing the incremental analysis

Taking Alternative A as the base alternative (lower investment)



Incremental cash flow diagram (B-A)

$PW_{(B-A)} (i^* \%) = 0$

$-1{,}50{,}000 + (60{,}000 - 73{,}800)\,(P/A, i^*, 8) + 3{,}50{,}000\,(P/F, i^*, 4)$

$= 0$

$i^*\% = 12.70 >$ MARR (10%)

Select Project B.

Example 6.9

Consider the Two mutually exclusive projects. Recommend the best project using the IRR method. MARR =15%.

EOY	A	B
0	- Rs. 12,500	- Rs. 15,000
1	- Rs. 5,000	- Rs. 4,000
2	- Rs. 5,000	- Rs. 4,000
3	-Rs.5,000+ Rs. 2000	- Rs. 4,000
4		- Rs. 4000+ Rs. 1,500

Solution

Since the Study period is LCM of 3 and 4 i.e. 12 years, we may compute the incremental cash flow over this 12-year period. As shown in figure (c), we subtract cash flows from Alternative A from Alternative B to form the increment of investment. (We want the first cash flow difference to be negative). Alternative A is repeated 4 times and alternative B is repeated 3 times

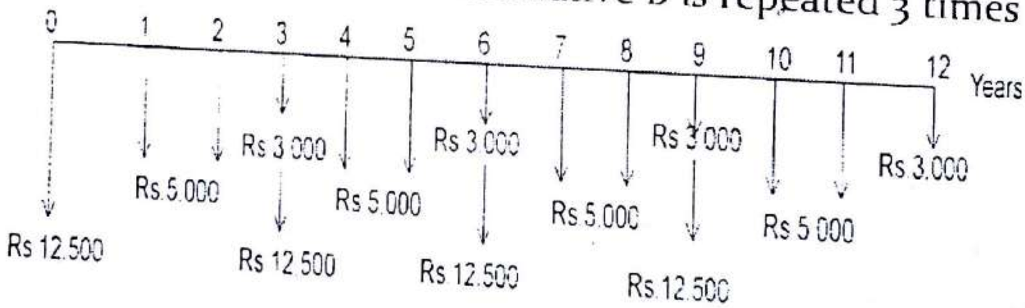


Fig (a): Cash flow diagram of Alternative A

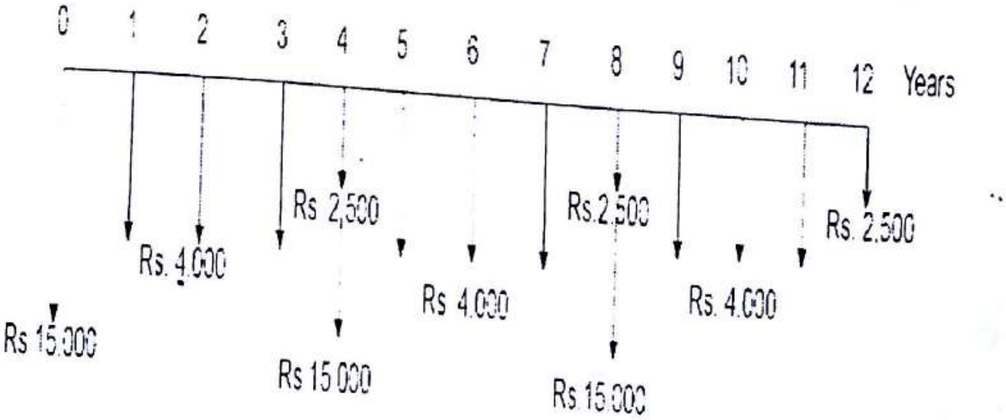


Fig (b): Cash flow diagram of Alternative B

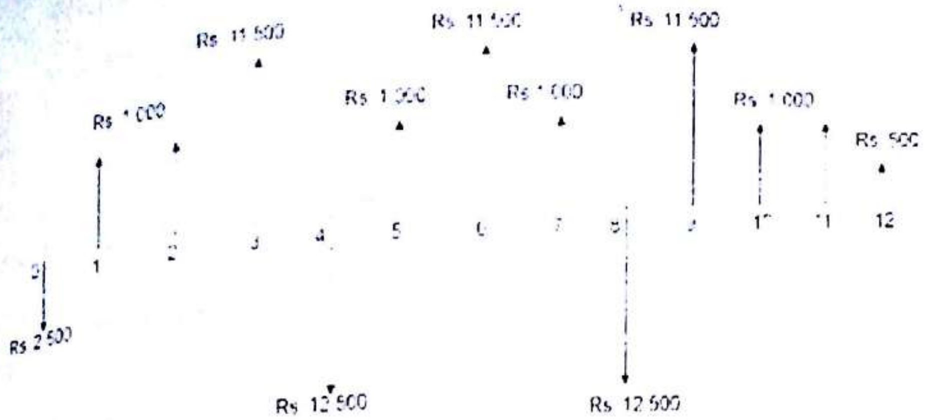


Fig (c): Incremental Cash flow
(Alternative B – Alternative A)

Here, five sign changes in the incremental cash flow, indicating non-simple incremental investment. This results in the multiple rate of return. Therefore we abandon the rate of return analysis and use the PW criterion.

PW (15%) $_{B\text{-}A}$ = -Rs.2, 500 + \$1,000(P/F, 15%, 1) +............. + Rs.500 (P/F, 15%, 12)

= **Rs.5, 123 > 0**

This indicates that PW (15%) $_B$ > PW (15%) $_A$, **select project B.**
The values of cash flow diagram is given in the table below

EOY	Projec t A (Rs)		Project B (Rs)		Project B – Project A (Rs)
0	-.12,500		-.15,000		-.2,500
1		- 5,000		- 4,000	.1,000
2		-.5,000		-.4,000	.1,000
3	-.12,500	-.3,000		-.4000	.11,500
4		-.5,000	- 15,000	-.2,500	- R12,500
5		-.5,000		- 4000	1,000
6	-.12,500	-.3,000		- 4,000	11,500
7		-.5000		- 4000	.1,000
8		-.5,000	-.15,000	-.2,500	- 12,500
9	- 12,500	-.3,000		-.4000	11,500

10		-.5,000		-.4,000	1,000
11		-.5,000		- 4,000	1,000
12		- 3,000		-.2,500	500

6.4 Capitalized Worth Method (CW)/ Capitalized Cost (CC)

Capitalized cost is the present worth of an alternative that will last "forever". It is the special case of PW criterion which is useful when the life of a proposed project is perpetual or the planning horizon is extremely long (say, 40 years or more). Many public sector projects such as bridges, waterway construction, irrigation systems, and hydroelectric dams are expected to generate benefits over an extended period of time (or forever). This criterion for evaluating and comparing the alternatives is useful in places where the repeatability assumption is applicable. In this section capitalized equivalent [CW (i)] method for evaluating such project is examined.

Let us consider the cash flow as shown in the figure below

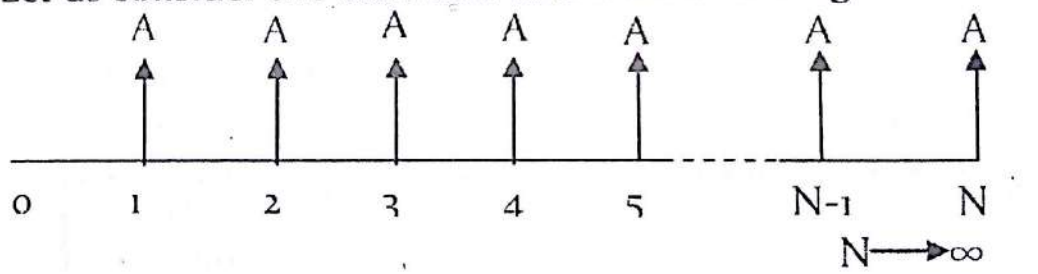


Fig: Equivalent present worth of an infinite cash flow series

The capitalized cost represents the amount of money that must be invested today to yield a certain return A at the end of each and every period forever, assuming interest rate i.

The formula to calculate CW is derived from the relation P = A (P/A, i, n), where N= ∞.

The equation of P using the P/A factor formula is

$$PW(i\%) = A\,[(1+i)^N - 1] / [i * (1+i)^N]$$

Divide the numerator and denominator by $(1+i)^N$

$PW(i\%) = A\{1/i - 1/(1+i)^N\}$

As N approaches ∞, the bracketed term becomes 1/i, and the symbol CW replaces PW or P.

$$CW(i\%) = A\{1/i - 1/\infty\} = A/i$$

$$CW(i\%) = (AW/i)(i\%)$$

Example 6.10

Assume infinite project life; recommend one of the following mutually exclusive projects (T.U.2064)

Project	A	B
Initial investment (Rs)	50,000	1,20,000
Salvage value (Rs)	10,000	10,000
Annual cost (Rs)	9,000	6,000
Useful life (years)	10	25
MARR	15%	

Solution

Calculate the AW of the both alternatives.

$AW_A(15\%) = -50,000\ (A/P, 15\%, 10) - 9000 + 10,000\ (A/F, 15\%, 10)$

$= -9,962.60 - 9,000 + 492.52$

$= Rs\ -18,470.08$

AC_A = **Rs. 18,470.08**

$AW_B(15\%) = -1,20,000\ (A/P, 15\%, 25) - 6000 + 10,000\ (A/F, 15\%, 25)$

$= -18563.92 - 6000 + 46.99$

$= Rs\ -24,516.93$

AC_B = **Rs. 24,516.93**

Divide the AW of both alternatives by interest rate, i.

$CW_A(15\%) = AW_A(15\%)/i = Rs\ -18,470.08/0.15 =$ **Rs – 1, 23,133.86**

$CW_B(15\%) = AW_B(15\%)/i = Rs\ -24,516.93/0.15 =$ **Rs – 1, 63,446.2**

$CW_A(15\%) < CW_B(15\%)$, **select project A**

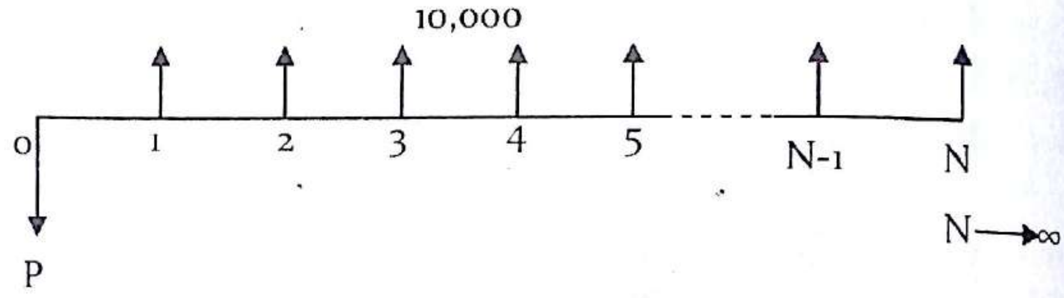
Example 6.11

How much Rupees should you deposit now in a saving account which gives 10% interest per year if you wish to draw Rs 10,000 per month for (i) 50 years (ii) continuously (infinite period).

Solution

Here, i= 10% per year and withdrawal is monthly. So we have to convert yearly interest rate to monthly interest rate

$i_{monthly} = (1+i_{year})^{1/12} - 1 = (1.1)^{1/12} - 1 = 0.797\%$



$P = A\{((1+i)^{N} - 1) / ((1+i)^{N} * i)\}$

(i) If N =(50 years *12) = 600 months

$P = 10{,}000\{((1+0.00797)^{600} - 1) / ((1+0.00797)^{600} * i)\}$

P = **Rs 12, 47, 086.35 (Ans)**

(ii) If $N \rightarrow \infty$

P = A/i = 10,000 /0.00797 = **Rs 12, 54, 705.144 (Ans)**

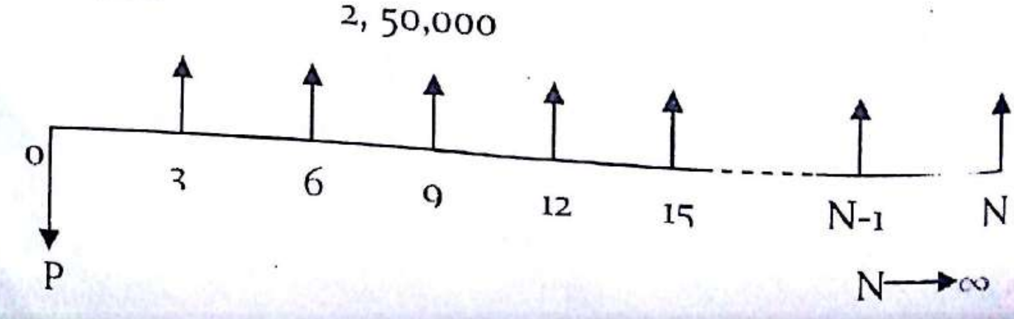
Here the difference is only Rs 7,618

What will you do?

Example 6.12

How much Rupees should you deposit now in a saving account which gives 8% interest per year to draw Rs 2, 50,000 at the end of 3rd year each?

Solution



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If $N \rightarrow \infty$, $P = A/i$

$A = 2,50,000$,

Interest rate at the end of 3rd year $(i_3) = (1+0.08)^3 - 1 = 0.2597$

$P = 2,50,000 / 0.2597 =$ **Rs. 9, 62,604.73 (Ans)**

Example 6.13

A selection is to be made between two structural designs. Because revenues do not exist (or can be assumed to be equal), only negative cash flow amounts (costs) and the market value at the end of useful life are estimated as follows.

Designs	Structure M	Structure N
Initial investment (Rs)	-12,000	-40,000
Salvage value (Rs)	0	10,000
Annual cost (Rs)	-2,200	-1,000
Useful life (years)	10	25
MARR	15%	

Using the repeatability assumption and the CW method of analysis, determine which structure is better?

Solution

Calculate the AW of the both alternatives

AW_M (15%) = -12,000 (A/P, 15%, 10) -2200 + 0

= **Rs -4,592**

AW_N (15%) = -40, 000 (A/P, 15%, 25) -1000 + 10,000 (A/F, 15%, 25)

= **Rs -7,141**

Divide the AW of both alternatives by interest rate, i.

CW_M (15%) = AW_M (15%)/i = Rs -4,592/0.15 = Rs – 30,613

CC_M(15%) = **Rs 30,613**

CW_N (15%) = AW_N (15%)/i = Rs -7,141/0.15 = Rs – 47,607

CC_N(15%) = **Rs • 47,607**

CW_M (15%) < CW_N (15%), **select project M**

6.5 Definition of Mutually Exclusive Investment Alternatives in terms of Combination of Projects

In this section, we will examine the process of deciding whether projects should be included in the capital budget. In particular, we will consider decision procedures that should be applied when we have to evaluate a set of multiple investments alternatives for which we have a limited capital budget. Here we distinguish a project from investment alternatives, which is a decision option. For a single project, we have two investment alternatives: to accept or reject the project. For two independent projects, we can have four investment alternatives: (1) to accept both projects, (2) to reject both projects (3) to accept only the first project and (4) to accept only the second project. To perform a proper capital budgeting analysis a firm must group all projects under consideration into decision alternatives. This grouping requires the firm to distinguish between projects that are independent of one another and those that are dependent on one another to formulate alternatives correctly.

Independent Project

An independent project is one that may be accepted or rejected without influencing the accept-reject decision of another independent project. For example, the purchase of a machine, office furniture and truck constitutes three independent projects. Only projects that are economically independent of one another can be evaluated separately.

Dependent projects

In many decision problems, several investment projects are related to one another such that the acceptance or rejection of one project influences the acceptance of others. The two such types of dependencies are as follows:

Contingent: Two or more projects are said to be contingent if the acceptance of one requires the acceptance of another. For example, the purchase of a computer printer is dependent upon the purchase of a computer, but computer may be purchased without considering the purchase of the printer.

Mutually exclusive: When there are several alternatives to achieve the same objectives and we can choose only one of them then the alternatives are called mutually exclusive project.

Formulation of Mutually Exclusive Alternatives

1. If A, B are two independent projects then the mutually exclusive combinations is:

Mutually exclusive combination	A	B	Remarks
1	-	-	Do nothing
2	√	-	Accept A
3	-	√	Accept B
4	√	√	Accept both A & B

2. If A, B, C is three mutually exclusive alternatives then we can make the following combination.

Mutually exclusive combination	A	B	C	Remarks
1	-	-	-	Do nothing
2	√	-	-	Accept A
3	-	√	-	Accept B
4	-	-	√	Accept C

3. If A, B, C are three project where C is contingent on the acceptance of B and acceptance of B is contingent of acceptance of A. we can make the following combination.

Mutually exclusive combination	A	B	C	Remarks
1	-	-	-	Do nothing
2	√	-	-	Accept A

3	√	√	-	Accept A and B
4	√	√	√	Accept all

Do nothing option (DN)

Selection of the DN alternative means that the current approach is maintained; nothing new is initiated. If there is certainty that one of the defined alternatives be selected, do nothing is not considered an option.

Example 6.14

Engineering projects B1, B2, C1, C2 and D are being considered with cash flows estimated over four years are as shown in table below. Using the PW method and MARR = 10% per year, determine what combination of projects is best if the capital to be invested is (a) unlimited and (b) limited to Rs. 48000. The combination of project is

Project B1and Project B2 (mutually exclusive and independent of C set)

Project C1and Project C2 (mutually exclusive and contingent on the acceptance of B2

Project D (contingent on the acceptance of C1)

Cash Flows for End of Year (Rs.)					
Project	0	1	2	3	4
B1	-50000	20000	20000	20000	20000
B2	-30000	12000	12000	12000	12000
C1	-14000	4000	4000	4000	4000
C2	-15000	5000	5000	5000	5000
D	-10000	6000	6000	6000	6000

Solution

Calculate the PW of all alternative

PW_{B1} = -50,000 + 20,000 (P/A, 10%, 4) = Rs.13, 400

PW_{B2} = -30,000 + 12,000 (P/A, 10%, 4) = Rs.8, 038

PW_{C_1} = -14,000 + 4,000 (P/A, 10%, 4) = - Rs.1, 300 (not been eliminated from consideration because project d is contingent on it)

PW_{C_2} = -15,000 + 5,000 (P/A, 10%, 4) = Rs.849

PW_D = -10,000 + 6,000 (P/A, 10%, 4) = Rs.9, 019

Mutually exclusive project combination

Mutually exclusive combination	Project				
	B1	B2	C1	C2	D
1	0	0	0	0	0
2	1	0	0	0	0
3	0	1	0	0	0
4	0	1	1	0	0
5	0	1	0	1	0
6	0	1	1	0	1

Mutually exclusive combination	Cash Flows for End of Year (Rs.)					Invested capital (Rs)	PW (10%) (Rs.)
	0	1	2	3	4		
1	0	0	0	0	0	0	0
2	-50,000	20,000	20,000	20,000	20,000	-50,000	13,400
3	-30,000	12,000	12,000	12,000	12,000	-30,000	8,038
4	-44,000	16,000	16,000	16,000	16,000	-44,000	6,738
5	-45,000	17,000	17,000	17,000	17,000	-45,000	8,887
6	-54,000	22,000	22,000	22,000	22,000	-54,000	15,757

Decision

1. Mutually exclusive combination 6 is the best combination which PW is Rs. 15757 when the capital is unlimited.
2. Mutually exclusive combination 6, 2 are not feasible if capital is limited to Rs. 48000, of the remaining combinations, combination 5 has the highest present worth of Rs. 8887.

Some Solved examples

1. From the following four mutually exclusive projects, recommend the best one using Payback period, ERR and BCR methods. The study period is 5 years and MARR= €= 15% (TU, IOE, 2065)

Project	A	B	C	D
Initial Investment	5,00,000	4,00,000	7,00,000	6,00,000
Net Annual Revenue	1,25,000	1,10,000	1,70,000	1,35,000

Salvage value is 20% of the initial investment

Solution

Using the Payback period method

Project A

Period	Cash flow	PW of net cash flow (i=15%)	Cumulative Cash Flow
0	-5,00,000	-5,00,000	-5,00,000
1	1,25,000	1,08,700	-3,91,300
2	1,25,000	94,512.5	-2,96,787.5
3	1,25,000	82,187.5	-2,14,600
4	1,25,000	71,475	-1,43,125
5	1,25,000+ 1,00,000	1,11,870	-31,225

Here the cumulative balance doesn't changes into positive in 5 years and payback period is more than 5 years.

Project B

Period	Cash flow	PW of net cash flow (i=20%)	Cumulative Cash Flow

0	-4,00,000	-4,00,000	-4,00,000
1	1,10,000	95,656	-3,04,344
2	1,10,000	83,171	-2,21,173
3	1,10,000	72,325	-1,48,848
4	1,10,000	62,898	-85,950
5	1,10,000+ 80,000	94,468	+8,518

Here the cumulative balance turns positive in the year 5. So payback period lies between year 4 and 5. By interpolation we get 4.90 years

Project C

Period	Cash flow	PW of net cash flow (i=20%)	Cumulative Cash Flow
0	-7,00,000	-700000	-700000
1	1,70,000	147832	-552168
2	1,70,000	128537	-423631
3	1,70,000	111775	-311856
4	1,70,000	97,206	-2,14,650
5	1,70,000+ 1,40,000	1,54,132	-60,518

Here the cumulative balance doesn't changes into positive in 5 years and payback period is more than 5 years.

Project D

Period	Cash flow	PW of net cash flow (i=20%)	Cumulative Cash Flow
0	-6,00,000	-6,00,000	-6,00,000
1	1,35,000	1,17,396	-4,82,604
2	1,35,000	1,02,073.5	-3,80,530.5
3	1,35,000	88,762.5	-2,91,768

4	1,35,000	77,193	-2,14,575
5	1,35,000 + 1,20,000	1,26,786	-87,789

Here the cumulative balance doesn't changes into positive in 5 years and payback period is more than 5 years.

Decision: Choose Project B which gives the pay back period of 4.90 years

Using the ERR method

Project A

(a) Convert all the cash outflows to present time at 15%, i.e. 500000

(b) Convert all the cash inflows to future time at 15%

125000 (F/A, 15%, 5) + 0.2*500000 = 9, 42,800

(c) Equate the two equations

500000 (F/P, i*, 5) = 9, 42,800

$(1+i^*)^5 = 1.8856$

$1+i^* = 1.1352$, $i^* = 13.52\% <$ MARR (15%), **Not justified.**

Project B

(a) Convert all the cash outflows to present time at 15%, i.e. 4,00,000

(b) Convert all the cash inflows to future time at 15%

1, 15,000 (F/A, 15%, 5) + 0.2*4, 00,000 = 85537

(c) Equate the two equations

4, 00,000 (F/P, i*, 5) = 8, 55,376

$(1+i^*)^5 = 2.138$

$1+i^* = 1.164$, $i^* = 16.4\% >$ MARR (15%), **Justified.**

Project C

(a) Convert all the cash outflows to present time at 15% i.e. 7,00,000

(b) Convert all the cash inflows to future time at 15%

1, 70,000 (F/A, 15%, 5) + 0.2*7, 00,000 = 12, 86,208

(c) Equate the two equations

7, 00,000 (F/P, i*, 5) = 12, 86,208

$(1+i^*)^5 = 1.837$

1+i* = 1.129, i* = 12.9% < MARR (15%), **Not justified.**

Project D

(a) Convert all the cash outflows to present time at 15% i.e. 6,00,000

(b) Convert all the cash inflows to future time at 15%

1, 35,000 (F/A, 15%, 5) + 0.2*6, 00,000 = 10, 30,224

(c) Equate the two equations

6, 00,000 (F/P, i*, 5) = 10, 30,224

$(1+i^*)^5 = 1.717$

1+i* = 1.114, i* = 11.4% < MARR (15%), **Not justified.**

Decision: Among the entire project, only project B is justified and it is chosen.

Using B/C ratio method

Project A

PW (15%) $_{Benefits}$ = 1, 25,000 (P/A, 15%, 5) + 0.2*5, 00,000 (P/F, 15%, 5) = 4, 68,745

PW (15%) $_{costs}$ = 5, 00,000

B/C ratio = PW (15%) $_{Benefits}$ / PW (15%) $_{costs}$ = 4, 68,745 /5, 00,000 = 0.94<1 **Not justified**

Project B

PW (15%) $_{Benefits}$ = 1, 10,000 (P/A, 15%, 5) + 0.2*4, 00,000 (P/F, 15%, 5) = 4, 08,518

PW (15%) $_{costs}$ = 4, 00,000

B/C ratio = PW (15%) $_{Benefits}$ / PW (15%) $_{costs}$ = 4, 08,518 /4, 00,000 = 1.02 >1 **Justified**

Project C

PW (15%) $_{Benefits}$ = 1, 70,000 (P/A, 15%, 5) + 0.2*7, 00,000 (P/F, 15%, 5) = 6, 39,482

PW (15%) $_{costs}$ = 7, 00,000

B/C ratio = PW (15%) $_{Benefits}$ / PW (15%) $_{costs}$ = 6, 39,482 /7, 00,000 = 0.913 < 1 **Not justified**

Project D

PW (15%) $_{Benefits}$ = 1, 35,000 (P/A, 15%, 5) + 0.2*6, 00,000 (P/F, 15%, 5) = 5, 12,211

PW (15%) $_{costs}$ = 6, 00,000,

B/C ratio = PW (15%) $_{Benefits}$ / PW (15%) $_{costs}$ = 5, 12,211 /600000 = 0.85 < 1 **Not Justified**

Decision: Among the entire project, only project B is justified and it is chosen. Incremental Analysis is not necessary.

2. Nepal Airlines is planning to purchase a Jet Plane. The estimate on two types of plane under consideration is: (TU, IOE 2065)

Project	Plane A	Plane B
First Investment Cost	25,00,00,000	30,00,00,000
Annual O&M	1,50,00,000	1,00,00,000
Useful Life	4 years	6 years
Salvage Value	5,00,00,000	6,00,00,000
MARR	12%	

Which plane is the best one if it is believed that the plane will be used for (a) 4 years (b) infinite period?

Solution

(a) When the plane is used for 4 years

No adjustment is required for plane A

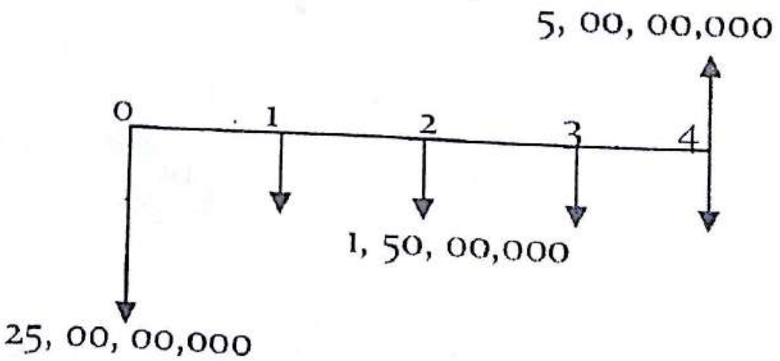


Fig: Cash flow diagram of plane A

Using PW formulation

PW (12%) = -25, 00, 00,000 – 1, 50, 00,000 (P/A, 12%, 4) + 5, 00, 00,000 (P/F, 12%, 4)

= - 250000000 – 45,55,9,500 + 31775000

= - 26, 37, 84,500

Study period of Plane B (4 years) < useful life (6 years),

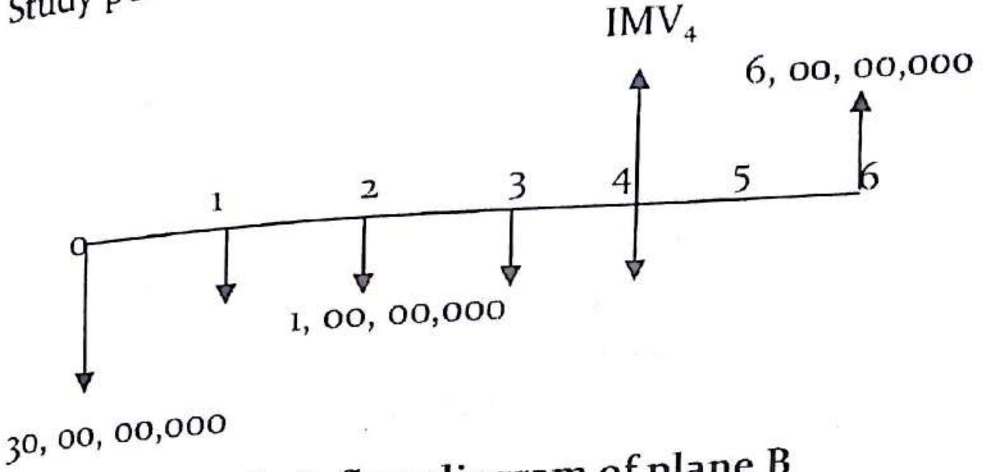


Fig: Cash flow diagram of plane B

Applying Imputed Market Value Technique

CR (12%) = 30, 00, 00, 000 (A/P, 12%, 6) – 6, 00, 00,000 (A/F, 12%, 6)

= 7, 29, 69,000 – 73, 93,800

= 6, 55, 75,200

PW of Remaining CR (at the end of 4 year)

PW_{CR} (12%) = 6, 55, 75,200 (P/A, 12%, 2)

= 11, 08, 28,645.5

PW of Market Value (at the end of 4 year)

PW_{MV} (12%) = 6, 00, 00,000 (P/F, 12%, 2) = 47,83,2,000

Imputed Market value (at the end of 4 years)

PW_{CR} (12%) + PW_{MV} (12%) = 11, 08, 28,645.5 + 47,83,2,000

= 15, 86, 60,645.5

PW (12%) = -30, 00, 00,000 -1, 00, 00,000 (P/A, 12%, 4) + 15, 86, 60,645 (P/F, 12%, 4)

= -30, 00, 00,000 – 3, 03, 73,000 + 10, 08, 28,839.9

= -22, 95, 44,160.1

In this case, cost only is considered and it may be considered that benefit derived from both the options is same. Hence the option with lower cost to be chosen i.e. **plane B is preferred.**

(b) When the plane is used for infinite period

AW_A = -25,00,00,000 (A/P, 12%, 4) -1,50,00,000 + 5,00,00,000 (A/F, 12%,4)

= -82, 30, 1,656.5 – 1. 50, 00,000 +10, 46, 2,074.98

= -86, 83, 9,581.52

AW_B = -30,00,00,000 (A/P, 12%, 6) -1,00,00,000 + 6,00,00,000 (A/F, 12%,6)

= -72,95,1,324.71 – 1,00,00,000 + 73,93,715.35

= -7,55,57,609.37

CW_A = AW_A/i = -86,83,9,581.52/0.12

= -72, 36, 6, 3179.3

CC_A = 72, 36, 6, 3179.3

CW_B = AW_B/i = -75,55,7,609.37/0.12

= -62, 96, 46,744.7

CC_B = 62, 96, 46,744.7Here $CC_B < CC_A$ i.e. **Plane B is chosen.**

3. By using PW or FW method (using repeatability) method select the best project **(TU, IOE, 2064)**

Alternative	A	B
First Investment Cost (Rs)	4,00,000	6,00,000
Annual Revenues (Rs)	30,000	35,000
Annual O&M(Rs)	3,000	4,000
Useful Life (years)	6	8
Salvage Value(Rs)	4,000	7,000
MARR	12%	

Solution

Study Period: LCM of 6 and 8 = 24 years

Alternative A is repeated 4 times

A1	A2	A3	A4

Using PW formulation

PW$_A$ (15%) = - 4, 00,000 - 4, 00,000 (P/F, 15%, 6) - 4, 00,000 (P/F, 15%, 12) - 4, 00,000 (P/F, 15%, 18) + (30,000 – 3, 000) (P/A, 15%, 24) + 4,000 (P/F, 15%, 6) + 4,000 (P/F, 15%, 12) +4,000 (P/F, 15%, 18) + 4,000 (P/F, 15%, 24)

= - 4, 00,000 {1+ (1.15) $^{-6}$ + (1.15) $^{-12}$ + (1.15) $^{-18}$} + (30,000 – 3, 000) {(1+0.15) ^{24} – 1/ (1.15 ^{24} *0.15)} + 4,000{(1.15) $^{-6}$ + (1.15) $^{-12}$ + (1.15) $^{-18}$ + (1.15) $^{-24}$}

= - 4, 00,000 – 1, 72,920 – 74,760 – 32,320 + 1, 73,712.6 + 1,729.2 + 747.6 + 323.2 + 139.6

= Rs -5, 03,347.8

Alternative B is repeated 3 times

B1	B2	B3

PW$_B$ (15%) = -6, 00,000 – 6, 00,000 (P/F, 15%, 8) - 6, 00,000 (P/F, 15%, 16) + (35,000 - 4,000) (P/A, 15%, 24) + 7,000 (P/F, 15%, 8) + 7,000 (P/F, 15%, 16) + 7000 (P/F, 15%, 24)

= -6, 00,000 {1+ (1.15)$^{-8}$ + (1.15)$^{-16}$} + (35,000 -4,000) {(1+0.15) ^{24} – 1/ (1.15 ^{24} *0.15)} + 7,000{(1.15)$^{-8}$ + (1.15)$^{-16}$ + (1.15)$^{-24}$

= -6, 00,000 – 1, 96,140 – 64,140 + 1, 99,447.8 + 2,288.3 + 748.3 + 244.3

= **Rs – 6, 57,551.3**

Both the alternatives have lesser negative value and both are not preferable. Even if decision is to be made than we go for lesser negative value i.e. **Alternative A is chosen.**

4. Solve by the capitalized worth method and select (TU, IOE, 2064)

Project	A	B
Investment (Rs)	50, 000	1,20,000
Annual revenues (Rs)	10,000	10,000
Annual Cost (Rs)	9,000	6,000
Useful life (year	10	25
MARR	15%	15%

Solution

Calculate the annual equivalent of both the alternatives

AW_A (15%) = -50,000 (A/P, 15%, 10) +10,000 – 9,000

= - 8,962.5

AW_A (15%) = - 1, 20,000 (A/P, 15%, 25) + 10,000 -6,000

= -14,564

CW_A (15%) = AW_A (15%)/i = - 8962.5/ 0.15

= -59,750

CW_B (15%) = AW_B (15%)/i = -14564/0.15

= -97,093.33

Select project A (least negative value)

5. Government of Nepal started four projects A, B, C and D for the promotion of information technology in Nepal. The estimated cash flow over 10 year as shown in the table. The capital investment budget is limited to 90,000 and the MARR is 10% per year. **(P.U. 2005)**

Project	A	B	C	D
Initial investment	32,000	25,000	72,000	80,000
Annual revenues	7,000	5,000	12,000	16,000
Market Value	4,000	3,000	5,000	6,000
B&C	Mutually Exclusive			
D	Dependent on acceptance of C			
A	Dependent on acceptance of B			

Recommend which investment alternative should be selected? Use FW method.

Solution

Calculating the FW of each alternatives

FW (10%) A = -32,000 (F/P, 10%, 10) + 7,000 (F/A, 10%, 10) + 4,000 = 32,563.4

FW (10%) B = -25,000 (F/P, 10%, 10) + 5,000 (F/A, 10%, 10) + 3,000 = 17,844.5

FW (10%) C = -72,000 (F/P, 10%, 10) + 12,000 (F/A, 10%, 10) +5,000 = 9,502.4

FW (10%) D = -80,000 (F/P, 10%, 10) + 16,000 (F/A, 10%, 10) + 6,000 = 53,502.4

Combination	A	B	C	D	Remark
1	0	0	0	0	Do nothing
2	1	1	0	0	Select A and B
3	0	1	0	0	Select B
4	0	0	1	0	Select C
5	0	0	1	1	Select C and D

Combination	FW (10%)	Invested Capital
1	0	0
2	50,407.9	57,000
3	17,844.5	25,000
4	9502.4	72,000
5	63004.8	1,52,000

Decision

- If the capital is limited to 90,000, combination 5 is rejected because its capital exceeds 90,000.
- Among the remaining three combinations, combination 2 is the best because it has the highest future worth of 50,407.9.

6. Select by incremental rate of return method for a project. **(TU, IOE, 2064)**

Project	A	B	C	D	E
Investment cost (Rs)	900	1,500	2,500	4,000	5,000
Net annual revenue	150	270	400	920	1,100
MARR	10%	10%	10%	10%	10%
Life in years	5	5	5	5	5
SV	0	0	0	0	0

Solution

1. Calculate the IRR of the all project using PW formulation

PW_A (i* %) = -900 + 150 (P/A, i*%, 5) =0

or, -900 + 150 {((1+i*) ^{5}-1)/ ((1+i*) ^{5}*i*)} = 0

IRR = -5.78 % < MARR (not justified)

PW_B (i* %) = -1,500 + 270 (P/A, i*%, 5) = 0

or, -1,500 + 270 {((1+i*) ^{5}-1)/ ((1+i*) ^{5}*i*)} = 0

IRR= -3.4% < MARR (not justified)

PW_C (i* %) = -2,500 + 400 (P/A, i*%, 5) = 0

or, -2,500 + 400 {((1+i*) ^{5}-1)/ ((1+i*) ^{5}*i*)} = 0

IRR = -7% < MARR (not justified)

PW_D(i* %) = -4,000 + 920 (P/A,i*%,5) = 0

or, - 4,000 + 920 {((1+i*) ^{5}-1)/ ((1+i*) ^{5}*i*)} = 0

IRR = 4.85% < MARR (not justified)

PW_E(i* %) = -5,000 + 1,100 (P/A,i*%,5) = 0

or, -5,000 + 1,100 {((1+i*) ^{5}-1)/ ((1+i*) ^{5}*i*)} = 0

IRR = 3.26 %< MARR (not justified)

Here, none of the project is justified. No incremental analysis is necessary. Neither of the projects is selected.

7. Use IRR method to select best project

Project	A	B	C	D	E
Investment (Rs)	90,000	1,15,000	2,50,000	4,00,000	5,00,000

Revenues less cost (Rs)	15,000	27,000	40,000	90,000	1,40,000

Solution

Let us assume the life of all project as 10 years and MARR =10%

1. **Calculate the IRR of each project using PW formulation**

IRR_A $= -90{,}000 + 15{,}000 \{((1+i^*)^{10} - 1) / (1+i^*)^{10} * i\} = 0$
$= 10.56\%$

IRR_B $= -1{,}15{,}000 + 27{,}000 \{((1+i^*)^{10} - 1) / (1+i^*)^{10} * i\} = 0$
$= 19.54\%$

IRR_C $= -250{,}000 + 40{,}000 \{((1+i^*)^{10} - 1) / (1+i^*)^{10} * i\} = 0$
$= 9.61\%$

IRR_D $= -400{,}000 + 90{,}000 \{((1+i^*)^{10} - 1) / (1+i^*)^{10} * i\} = 0$
$= 18.31\%$

IRR_E $= -500{,}000 + 140{,}000 \{((1+i^*)^{10} - 1) / (1+i^*)^{10} * i\} = 0$, $IRR_A = 25\%$

2. **Compare with MARR = 10%**

IRR_A (10.56%) > MARR (10%), (accepted)
IRR_B (19.54%) > MARR (10%), (accepted)
IRR_C (9.61%) < MARR (10%), (rejected)
IRR_D (18.31%) > MARR (10%), (accepted)
IRR_E (25%) > MARR (10%) , (accepted)

3. Out of the remaining alternative (A, B, D,E), Perform the incremental analysis, taking the base alternative as A (least investment project) and comparing with next highest alternative, i.e. B

A (base alternative)	Incremental investment (B-A)	Incremental Cash flow	Incremental IRR	Is increment justifie

				d
-90,000	-25,000	12,000	46.98%	Yes

Here project A is left out and we select project B

Taking project B as base alternative and comparing with alternative C (next highest alternative)

B (base alternative)	Incremental investment (C-B)	Incremental Cash flow	Incremental IRR	Is increment justified
-1,15,000	-1,35,000	13,000	-0.68%	No

Here project C is left out and we go for project B

Taking project B as base alternative and comparing with alternative D (next highest alternative)

B (base alternative)	Incremental investment (D-B)	Incremental Cash flow	Incremental IRR	Is increment justified
-1,15,000	-2,85,000	63,000	17.81%	Yes

Here project B is left out and we go for project D

Taking project D as base alternative and comparing with alternative E (next highest alternative)

D (base alternative)	Incremental investment (E-D)	Incremental Cash flow	Incremental IRR	Is increment justified
-4,15,000	-1,00,000	50,000	49.08%	Yes

Hence Select Project E

8. Select the best project using ERR method. MARR =18% and ε=12% **(TU, IOE, 2066)**

Year	0	1	2	3	4	5
Project A	-40,000	-38,000	+35,000	+35,000	+35,000	+35,000
Project B	-60,000	+25,000	+40,000	-50,000	+50,000	+75,000

Solution

Calculate the ERR of both projects

Project A

Step 1: discount all cash outflows to present

40,000 + 38,000 (P/F, 12%, 1) = 73,930.2

Step 2: compound all cash inflows to future

35,000 (F/A, 12%, 4) = 1, 67,275.5

Step 3: establish the equivalence of two equations

73,930.2 (F/P, i', 5) = 1, 67,275.5

73930.2 $(1+i')^5$ = 1,67,275.5

$(1+i')^5$ = 2.2626

i' = 17.73 % < MARR (18%), **unacceptable**

Project B

Step 1: discount all cash outflows to present

60,000 + 50,000 (P/F, 12%, 3) = 95590

Step 2: compound all cash inflows to future

25,000 (F/P, 12%, 4) + 40,000 (F/P, 12%, 3) + 50,000 (F/P, 12%, 1) +75,000 = 2, 26,533.5

Step 3: establish the equivalence of two equations

95,590 (F/P, i', 5) = 2, 26,533.5

95,590 $(1+i')^5$ = 2,26,533.5

$(1+i')^5$ = 2.36

i' = 0.1883 = 18.83% >MARR (18%), **acceptable**

Here, project A is less than MARR and not feasible and Project B is greater than MARR and feasible. So no comparison is required. **Hence project B is selected.**

9. Three mutually exclusive alternatives are to be compared by the rate of return method and are describe below. MARR is 10%, salvage value is 20% of the first cost. Which option has the highest IRR and what it is? Recommend the best alternative. **(TU,IOE,2069)**

	X	Y	Z
First Cost (Rs)	70,000	60,000	100,000
Annual Income (Rs)	15000	10,000	18000
Economic life, years	8	8	8

Solution

Using the PW formulation

Salvage Value = 0.2* 70,000 = 14,000

IRR of X

$PW(i^* \%) = 0$

$PW_{inflow} - PW_{outflow} = 0$

$15000\,(P/A, i^*\%, 8) + 14000\,(P/F, i^*\%, 8) - 70000 = 0$

$15000 \{((1+i^*)^8 - i^*) / ((1+i^*)^5 * i)\} + 14000\,(1+i^*)^{-8} - 70{,}000 = 0$

Solving by trial and error,

$i^* = 0.1576 = 15.76\% > MARR$ (accepted)

Salvage Value = 0.2* 60,000 = 12,000

IRR of Y

$PW(i^* \%) = 0$

$PW_{inflow} - PW_{outflow} = 0$

$10000\,(P/A, i^*\%, 8) + 12000\,(P/F, i^*\%, 8) - 60000 = 0$

$10000 \{((1+i^*)^8 - i^*) / ((1+i^*)^8 * i)\} + 12000\,(1+i^*)^{-8} - 60{,}000 = 0$

Solving by trial and error,

$i^* = 0.0955 = 9.55\% < MARR$ (rejected)

Salvage Value = 0.2* 100,000 = 20,000

IRR of Z

$PW(i^* \%) = 0$

$PW_{inflow} - PW_{outflow} = 0$

$18000\,(P/A, i^*\%, 8) + 20000\,(P/F, i^*\%, 8) - 100000 = 0$

$18000 \{((1+i^*)^8 - i^*) / ((1+i^*)^8 * i)\} + 38000\,(1+i^*)^{-8} - 100{,}000 = 0$

Solving by trial and error,

$i^* = 0.1133 = 11.33\% > MARR$ (accepted)

Option X has the highest IRR of 15.76%

Performing the incremental analysis between option X and option Z

Choosing option X as base alternative and developing incremental cash flow.

EOY	Option X	Option Z – Option X
0	-70000	-30000
1	15000	3000
2	15000	3000
3	15000	3000
4	15000	3000
5	15000	3000
6	15000	3000
7	15000	3000
8	29000	9000
Incremental IRR	15.76%	-2.93% (not justified)

Since incremental IRR is not justified, the extra investment of 30000 in project Z is not feasible. Therefore project X is selected.

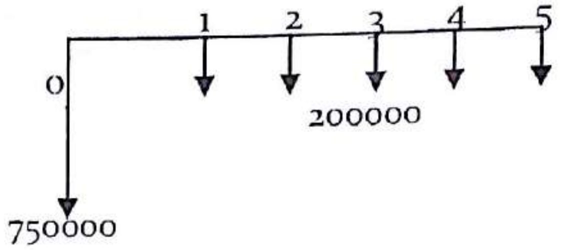
10. Two types of power converters, alpha and beta are under consideration for a specific application. An economic comparison is to be made at an interest rate of 12% and the following cost estimates have been obtained. Select the best option by calculating present worth of both the project if it will be operated for 4 years only. (TU,IOE,2068)

	Alpha	Beta
Purchase price (Rs)	750000	2000000
Annual operating cost (Rs)	200000	100000
Salvage Value (Rs)	0	400000
Estimated service life	5 years	9 years

Solution

Her both the options Alpha and Beta have the useful life greater than its study period. Use co terminated assumption and the imputed market value is calculated for the both.

Alpha



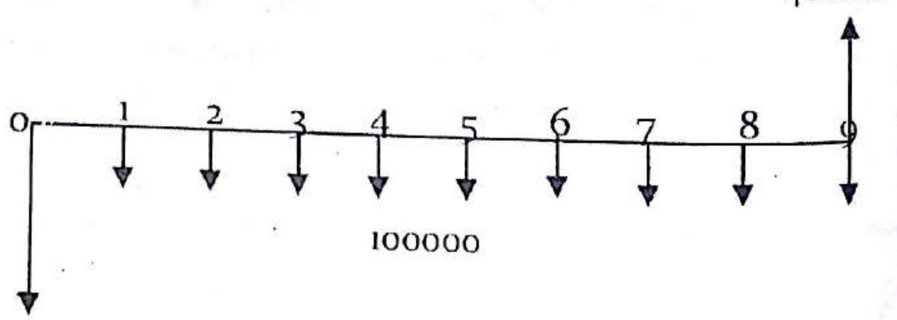
$$CR(12\%) = 750000\,(A/P, 12\%, 5) - 0$$
$$= 750000 * 0.2774 - 0 = 208050$$

$$PW(12\%)_{CR} = 208050\,(P/F, 12\%, 1)$$
$$= 208050 * 0.8929 = 185767.85$$

$$IMV_4 = PW_{CR} + PW_{MV}$$
$$= 185767.85 + 0 = 185767.85$$

$$PW(12\%) = -750000 - 200000\,(P/A, 12\%, 4) + 185767.85\,(P/F, 12\%, 4)$$
$$= -750000 - 200000 * 3.0373 + 185767.85 * 0.6355$$
$$= -1239404.5$$

Beta



$$CR(12\%) = 2000000\,(A/P, 12\%, 9) - 400000\,(A/F, 12\%, 9)$$
$$= 2000000 * 0.1877 - 400000 * 0.0677$$
$$= 348320$$

$$PW(12\%)_{CR} = 208050\,(P/A, 12\%, 5)$$

$= 348320 * 3.6048 = 1255623.93$

$PW(12\%)_{MV} = 400000\ (P/F, 12\%, 5)$

$= 400000 * 0.5674 = 226960$

$IMV_4 = PW_{CR} + PW_{MV}$

$= 1255623.93 + 226960 = 1482583.93$

$PW(12\%) = -2000000 - 100000\ (P/A, 12\%, 4) + 1482583.93\ (P/F, 12\%, 4)$

$= -2000000 - 100000 * 3.0373 + 1482583.93 * 0.6355$

$= -1361547.912$

Alpha has the least negative value and is the best option.

Review Questions

1. Explain Independent, Mutually exclusive and contingent project with suitable example.
2. Explain Repeatability and co terminated assumption?
3. Why incremental analysis is preferred in IRR method while comparing mutually exclusive projects?
4. Explain steps for incremental analysis.
5. What is capitalized worth method and where is it preferable?

Exercises

6. Consider the following two mutually exclusive investment projects. Assume MARR = 15%. Which project would be selected based on PW criterion?

	Project Cash Flow	
EOY	A	B
0	-Rs 3,000	-Rs 8000
1	400	11,500
2	7,000	400

7. Two alternative machines are being considered for a manufacturing process. Machine A has a first cost of Rs. 75,200 and its salvage value at the end of 6 years of

STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- Concept of Depreciation.
- The terminologies used in depreciation
- Book depreciation and tax depreciation
- Corporate tax and after tax cash flow

7.1 Meaning of Depreciation

Depreciation is the decrease in the value of physical properties with the passage of time and use. In economic analysis, value may refer to either market value or value to the owner. A non cash expense that reduces the value of an asset as a result of wear and tear or age and obsolescence. Most assets lose their value over time (in other words, they depreciate), and must be replaced once the end of their useful life is reached. It is an accounting concept that establishes an annual deduction against before tax income such that the effect of time and use on asset's value can be reflected in a firm's financial statements.

Depreciation charges are a convenient mechanism for recovering the capital that is invested. As such, when the time comes to replace an asset, funds will be available to do so (unless prices have increased over the asset's life). Also, treating depreciation charges as expenses allows one to incorporate such charges in the cost of production and ensure that prices are sufficient to recover invested capital. In general, assets can only be depreciated if they meet the following basic requirements:

- The property must be used for business purpose to produce income.
- The property must have a useful life that can be determined and this life must be longer than one year.

- The property must be an asset that decays, gets used up, wears out, becomes obsolete, or loses value to the owner from natural causes.

Engineers need to learn about depreciation because their design decisions can affect the way investments and annual operating costs are treated from an income tax perspective.

Basic requirements for property to be depreciated

- Depreciable property is property for which depreciation is allowed under government income tax laws and regulations.
- To determine if depreciable deductions can be taken, the classification of various types of property must be understood. In general, property is depreciable if it meets the following basic requirements:
  - It must be used in business or held to produce income.
  - It must have a determinable useful life and the life must be longer than one year.
  - It must be something that wears out, decays, become obsolete or loss value from natural causes.
- Depreciable property is classified as either tangible or intangible. Tangible property can be seen or touched and it includes two main types called personal property and real property.
- Personal property includes assets such as machinery, vehicles, equipment, furniture and other similar items.
- Real property is land. Land itself is not depreciable because it does not have determinable life.
- Intangible property is personal property such as copyright, patent, goodwill, and trademark.
- Depreciation stops when it is retired from service.

The loss in value, depreciation occurs for several reasons. It may be due to:

Use related physical loss

As something is used it wears out and must be replaced or repaired, but its value declines along with the "wear and tear" it suffers. Often, this aspect is measured with respect to units of production which in the car's case is measured in kilometers driven, hours of use of light bulb or pump etc.

Time related physical loss

Even if not used, things will deteriorate with time because of nature or other effects. Rusting in storage, chemical deterioration, paint drying in car, etc are some example of this loss. This loss is usually measured in units of time, such as a 10 year old car, a 40 year old sewage treatment plant etc.

Functionally related loss

The loss can occur without any physical changes for value can be lost over time as fashion changes, furniture, clothes, and new laws regarding pollution control etc.

Sudden Failure

It refers to the sudden or catastrophic loss in value due to technological characteristics inherent in the asset. However, this does include loss due to accident or misuse.

Depletion

Consumption of exhaustible natural resources to produce products or services is termed depletion. Removal of oil, timber, rock or minerals from a site decreases the value of holding. This decrease is compensated by a proportionate reduction in earnings derived from the resources.

Terminologies

1. **Cost basis or Unadjusted Cost**
   - Initial cost of the assets including purchase price, delivery and installation fees, and other depreciable direct costs incurred to prepare the asset for use.
2. **Book Value**
   - Represents the remaining, un-depreciated capital investment on the books after the total amount of depreciation charges to date have been subtracted from the basis.
   - The book value is usually determined at the end of each year.
3. **Recovery period**
   - It is the depreciable life 'n' of the assets in years.
4. **Salvage Value (SV)**
   - It is the estimated value of the property at the end of its useful life. It is the expected selling price of a property when the asset can no longer be used productively by its owner.
5. **Useful life**
   - It is the estimated or expected period of time that a property will be used in a trade or business or to produce income.
6. **Market Value (MV)**
   - The amount that will be paid by the willing buyer to a willing seller for a property where each has equal advantage and is under no compulsion to buy or sell. The MV approximates the present value of what will be received through ownership of the property, including the time value of money.

7.2 Methods of Depreciation

1. **Straight Line Method**

- It is the simplest and most often used depreciation method.
- It assumes that a constant amount is depreciated each year over the depreciable (useful life) of the asset
- In this method, a fixed or equal sum or amount is charged as the depreciation amount throughout the lifetime of the asset such that the accumulated sum at the end of the life of the asset is exactly equal to the purchase value of the cost, i.e. the value of the assets will become zero .

$$D_n = (I - S)/ N$$

Where,

D_n = Depreciation charge during year n

I = Cost of the assets including installation expenses

S = Salvage value at the end of useful life

N = Useful life

Book Value in a given year = *Cost Basis – total depreciation charges made to date*

Example 7.1

A machine costing Rs 11,000 is estimated to have a life of 10 years and the salvage value is estimated at Rs 1,000 at the end of the life. Determine the annual depreciation and annual rate of depreciation.

Solution

The annual depreciation = (11,000 – 1,000)/10 = Rs 1,000

Annual rate of depreciation = 1,000/ 10,000 = 0.1*100%

= 10% **Ans**

Example 7.2

Consider the following automobile data:

Cost of the asset, (I) = Rs 10,000

Useful life, (N) = 5 years

Estimated Salvage value (S) = Rs2, 000

Compute the annual depreciation allowance and the resulting book value using the straight line depreciation method?

Solution

Given: I = Rs10, 000

S = Rs2, 000

N = 5 years

The straight line depreciation rate is 1/5 or 20%. Therefore the annual depreciation charge is

D_n = (0.20) (Rs10, 000 - Rs2000) = Rs 1600

The asset would have the following book value during its useful life

n	B_{n-1} (Rs)	D_n (Rs)	B_n (Rs)
1	10,000	1600	8400
2	8400	1600	6800
3	6800	1600	5200
4	5200	1600	3600
5	3600	1600	2000

Where B_{n-1} and B_n represents the book value before depreciation and after depreciation

2. **Declining / Diminishing balance Method (DB)**

- Declining balance is also known as the fixed percentage or uniform percentage method.
- In this method, Book value is multiplied by the fixed rate

$$\alpha = (1/N) \text{ multiplier}$$

Example 7.2

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$$D_n = (0.20)\ (\text{Rs}10,000 - \text{Rs}2000) = \text{Rs } 1600$$

The asset would have the following book value during its useful life

n	B_{n-1} (Rs)	D_n (Rs)	B_n (Rs)
1	10,000	1600	8400
2	8400	1600	6800
3	6800	1600	5200
4	5200	1600	3600
5	3600	1600	2000

Where B_{n-1} and B_n represents the book value before depreciation and after depreciation

2. **Declining / Diminishing balance Method (DB)**

- Declining balance is also known as the fixed percentage or uniform percentage method.
- In this method, Book value is multiplied by the fixed rate

$$\alpha = (1/N)\ \text{multiplier}$$

- The most commonly used multipliers are double the straight line rate, for this reason it is called ***Double Declining Balance Method.***
- Suppose a business has an asset with Rs.1, 000 Original Cost, Rs100 Salvage Value, and 5 years useful life.
- First, calculate straight-line depreciation rate.
- Since the asset has 5 years useful life, the straight-line depreciation rate equals (100% / 5) 20% per year.
- With double-declining-balance method, as the name suggests, double that rate, or 40% depreciation rate is used.

We can determine depreciation charges for a given year, D_n as follows:

$$D_1 = \alpha I$$
$$D_2 = \alpha (I - D_1) = \alpha I (1 - \alpha)$$
$$D_3 = \alpha (I - D_1 - D_2) = \alpha I (1 - \alpha)^2$$

Thus for any year n, we have depreciation charge

$$D_n = \alpha I (1 - \alpha)^{n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Total Declining Balance (TDB) at the end of year n can be calculated as

$$TDB = D_1 + D_2 + D_3 + \ldots\ldots\ldots\ldots + D_n$$
$$= \alpha I + \alpha I (1 - \alpha) + \alpha I (1 - \alpha)^2 + \ldots\ldots\ldots\ldots + \alpha I (1 - \alpha)^{n-1}$$
$$= \alpha I \{1 + (1 - \alpha) (1 - \alpha)^2 + \ldots\ldots\ldots\ldots + (1 - \alpha)^n \ldots\ldots\ldots (2)$$

Multiplying by (1- α) we obtain

$$(1 - \alpha) TDB = \alpha I \{(1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)^3 + \ldots\ldots + (1 - \alpha)^n \ldots (3)$$

Subtracting equation (2) from equation (3)

$$\mathbf{TDB = I \{1 - (1 - \alpha)^n\}}$$

The book Value, B, at the end of each year will be the cost of the asset (I) minus the total depreciation at the end of N year.

$B_n = I - TDB$

$B_n = I - I\{1 - (1-\alpha)^n\}$

$B_n = I(1-\alpha)^n$

Example 7.3

Consider the following accounting information for a computer system.

Cost Basis of the asset, I, = Rs 10,000

Useful life, N, = 5 years

Estimated Salvage Value, S, = Rs778

Compute the annual depreciation allowances and the resulting book values using the double declining deprecation method.

Solution

- The book value at the beginning of the first year is Rs 10,000.
- The declining balance rate (α) is (1/5) *2 = 40%
- The depreciation deduction for the first year will be Rs 4000 (40%* $10,000 = Rs4000)
- The book value at the beginning of the second year is Rs 6,000 $\{I(1-\alpha)^n\}$ = Rs 10,000 $(1-0.4)^1$ =Rs 6,000}
- The depreciation deduction for the second year will be Rs 2,400 (40% * Rs 6,000 = Rs2,400)
- The book value at the beginning of the third year is Rs 6,000 { $I(1-\alpha)^2$ = Rs10,000 $(1-0.4)^2$ = Rs 3,600}

On continuing this process, we obtain the depreciation value and book value as shown in Table:

n	B_{n-1} (Rs)	D_n (Rs)	B_n (Rs)
1	10,000	4,000	6,000
2	6,000	2,400	3,600
3	3,600	1,440	2,160
4	2,160	8,64	1,296
5	1,296	5,18	7,78

Issues Regarding salvage Value

- Salvage value (S) must be estimated at the beginning of depreciation analysis.
- In previous example, the final Book Value (B_n) equals to the estimated salvage value, an occurrence that is rather unusual in the real world.
- When BN ≠ S, we have to make the adjustments in our depreciation analysis methods.
- Two cases Arises:

Case 1: When $B_N > S$

Case 2: When $B_N < S$

Case 1: When $B_N > S$

- When $B_N > S$, is the situation in which we have not depreciated the entire cost of the asset.
- To reduce the book value of an asset to its salvage value as quickly as possible, it can be done by *switching from declining balance (DB) to Straight line (SL).*
- The switch from DB to SL depreciation can take place in any of the 'n' years (optimal year).
- The switching rule is as follows:
  If depreciation by DB in any year is less than (or equal to) the depreciation by SL then we would switch to and remain with SL method for the duration of the project's depreciable life.
- The straight line depreciation is any year n is calculated by

$$D_n = \frac{\text{book value at the beginning of year n} - \text{salvage value}}{\text{Remaining useful life at beginning of year n}}$$

Example 7.4

Consider the following accounting information for a computer system.

Cost Basis of the asset, I, = Rs 10,000
Useful life, N, = 5 years
Salvage Value, SV, = Rs0
Declining balance Rate (α) = (1/5) *2 = 40%

Determine the optiimal time to switch from DB to SL depreciation and the resulting depreciation schedule.

Solution

- First, Computing the DDB depreciation for each year.

n	B_{n-1} (Rs)	D_n (Rs)	B_n (Rs)
1	10,000	4,000	6,000
2	6,000	2,400	3,600
3	3,600	1,440	2,160
4	2,160	8,64	1,296
5	1,296	5,18	7,78

Here the Book value is Rs778 at the end of Year 5 which is *Greater than zero.* Therefore we use the *switching fundamentals.*

- Second, we compute the SL depreciation for each year then compare SL to DDB depreciation for each year and use the decision rule for when to change.

If switch to SL Beginning of year	SL Depreciation	DDB Depreciation	Decision
1	(10000-0)/5 = 2000	<4000	Do not switch
2	(6000 -0)/4=1500	<2400	Do not switch
3	(3600 -0) /3 = 1200	<1440	Do not switch
4	(2160- 0) 2 = 1080	>864	Switch to SL

The optimal year is 4 in this situation

- Third, use the SL depreciation method for the remaining depreciable year from the optimal year.

- The depreciation schedule is

Year	DDB with switching to SL (Rs)	End of Year Book Value (Rs)
1	4000	6000
2	2400	3600
3	1440	2160
4	1080	1080
5	1080	0

Case 2: When $B_N < S$

- When $B_N < S$, we must re-adjust our analysis because tax law doesn't permit us to depreciate the asset below their salvage value.
- If the book value is lower than S, at any period, then the depreciation amount are adjusted so that $B_n = S$.

Example 7.5

Compute the double declining Balance (DDB) depreciation schedule for the data as follows:

Cost basis of the asset, I, = Rs 10,000
Useful life, N, = 5 years
Salvage Value, SV, = Rs 2,000
Declining balance Rate (α) (1/5) = 40%

Solution

End of Year	D_n (Rs)	B_n (Rs)
1	0.4 *10,000 = 4000	10,000 – 4000 = 6, 000
2	0.4*6000 = 2,400	6,000 – 2,400 = 3,600
3	0.4* 3,600 = 1,440	3,600 – 1440 = 2,160
4	0.4*2,160 = 864 > 160	2160 – 160 = 2,000
5	0	2,000 – 0 = 2,000

Here B_4 would be less than SV =Rs. 2000, if the full deduction (Rs 864) had been taken. Therefore we adjust D_4 to Rs 160, making B_4 = Rs. 2000. D_5 is zero and B_5 remains at Rs 2,000.

3. **Sum-of- Years' Digit (SOYD) Method**

- This method results in larger than straight line depreciation charges during the early years of an asset and smaller charges as the asset nears the end of its estimated useful life.
- Each year depreciation charge is compound as the remaining useful life at the beginning of the year divided by the sum of the years digits for the total useful life.
- In this method, the number 1, 2, 3...N are summed, where N is the estimated years of useful life.

$$SOYD = 1+2+3+ \dots\dots\dots + N = \{N(N+1)\}/2$$

- The depreciation each year is calculated as

$$D_n = \frac{\{N - n + 1\}(I - S)}{SOYD}$$

Example 7.5

Compute the SOYD depreciation schedule for the following:

Cost basis of the asset, I, = Rs 10,000

Useful life, N, = 5 years

Salvage Value, SV, = Rs 2,000

Solution

We first compute the sum – of – years' digits

SOYD = 1+2+3+4+5 = 15 or 5 (5+1) / 2 = 15

End of Year	D_n (Rs)	B_n (Rs)
1	5/15 (10,000 – 2,000) =2,667	10,000 – 2,667 = 7, 333

2	4/15 (10,000 – 2,000) =2,133	7, 333– 2,133 = 5,200
3	3/15 (10,000 – 2,000) =1,600	5,200– 1,600 = 3,600
4	2/15 (10,000 – 2,000) =1,067	3,600 – 1,067 = 2,533
5	1/15 (10,000 – 2,000) =5,33	2,533– 5,33 = 2,000

4. **Sinking fund method**

- In this method of depreciation, the book value decreases at increasing rates with respect to the life of the asset.
- The loss in value of asset (cost –salvage) is made available and the form of cumulative depreciation amount at the end of the asset by setting up an equal depreciation amount at the end of the each period during the lifetime of asset.
- The fixed sum depreciated at the end of every time period earns an interest at the rate of i% compounded annually.

Let,

P = first cost of the asset

F = salvage value

N = life of the asset

i = rate of return compounded annually (rate of interest)

A = the annual equivalent amount of depreciation charge

BV_k = the book value of the asset at the end of period k,

D_t = the depreciation charge at the end of the period t.

To find the annual equivalent amount (A) = (P-F) * (A/F, i%, N)

To find the depreciation charge (D_t) = (P-F) * (A/F, i%, N) * (F/P, i%, t-1)

To find the book value at the end of period t = P – (P-F) * (A/F, i%, N) (F/A, i%, t)

Example 7.6

Compute the depreciation charge and book value in each year using sinking fund method.

Cost of the asset = Rs 1, 00,000

Salvage value = Rs 20,000

Life of the asset (N) = 8 years

Interest rate (i) = 12%

Solution

To find A, the annual depreciation charge

A = (P-F) (A/F, 12%, 8) = (1, 00, 000 – 20, 000) (0.0813)

= Rs 6,504

Depreciation at the end of 1st year (D_1) = Rs 6,504

Depreciation at the end of 2nd year (D_2)

= 6,504 + (6,504 *0.12) = Rs 7284.48

Depreciation at the end of 3rd year (D_3)

= {6,504 + (6,504 + 7,284.48) *0.12} = Rs. 8,158.62

Depreciation at the end of 4th year (D_4)

= {6,504 + (6,504 + 7,284.48+8,158.62) *0.12}

= Rs. 9,137.65

These computations along with book values are as follows

EOY (t)	Fixed depreciation	Net depreciation(D_t) Rs	Book value (B_t) Rs
0	6, 504		1,00,000
1	6, 504	6, 504.00	93,496.00
2	6, 504	7,284.48	86,211.52
3	6, 504	8,158.62	78,052.90
4	6, 504	9,137.65	68,915.25
5	6, 504	10,234.17	58.681.08
6	6, 504	11,462.27	47,218.81
7	6, 504	12,837.74	34,381.07
8	6, 504	14,378.27	20,002.80

5. Modified Accelerated Cost Recovery System (MACRS)

- Historically, for the tax purposes as well as for accounting, an asset's depreciable life was determined by its estimated useful. i.e. an asset would be fully depreciated at approximately the end of its useful life.
- The MACRS scheme, totally abandon this practice and simpler guidelines were set which created several classes of assets, each with more or less arbitrary life called Recovery Period.
- The MARCS scheme includes 8 categories of assets: 3 year, 5 year, 7 year, 10 year, 15 year, 20 year, 27.5 year and 39 year.
- The salvage value of property is always treated as zero.

MACRS Property Classification

Recovery Period	Applicable Property
3 year	Special tools for manufacture of plastics products, metal products and motor vehicle
5 year	Automobiles, light trucks, equipment used of R&D, computerized telephone switching systems.
7 year	Manufacturing equipment, office furniture.
10 year	Vessels, barges, tugs
15 year	Waste water plants, telephone distribution plants
20 year	Municipal sewers, electrical power plants
27.5 year	Residential rental property
39 year	Nonresidential real property including elevators and escalators

Half year Convention

- The MACRS scheme uses the half-year convention i.e. it is assumed that all assets are placed in service at mid-year and they have zero salvage value.
- Only half year depreciation is allowed for the first year of the asset placed in service.
- With half year's depreciation being taken in service, a full year's depreciation is allowed in each of the remaining years of the assets recovery period.
- Finally the remaining half year depreciation in the year following the end of the recovery period.

Switching from Declining Balance to Straight Line Method

- The MACRS asset is depreciated initially by the Declining balance (DB) method and then Straight line method.
- It adopts the switching convention as discussed in the previous section.

MACRS Depreciation Rates

Year	3 year	5 year	7 year	10 year	15 year	20 year
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44,45	32.00	24.49	18.00	9.50	7.219
3	14.81	19.20	17.49	14.40	8.55	6.667
4	7.41	11.52	12.49	11.52	7.70	6.177
5		11.52	8.93	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55	5.90	4.888
8			4.46	6.56	5.90	4.522
9				6.55	5.91	4.462
10				6.55	5.90	4.461
11				3.28	5.91	4.462

12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

Example 7.6

A tax payer wants to place in service a Rs 10,000 asset that is assigned to the 5 year class. Compute the MACRS percentage and depreciation amounts for the asset.

Solution

MACRS deduction percentages, beginning with the first taxable year and ending with the 6th year are computed as follows.

Straight line rate = 1/5 = 0.20

Double declining balance rate (α) = 1/5 *200% = 40%

Under MACRS, salvage value = 0

Year	Calculation (%)	MACRS %	Decision
1	½ year DDB dep. = (0.5) (0.4) (100%)	20%	
2	DDB dep. = (0.4) (100% -20%) SL dep. (1/4.5) (100% -20%)	32% 17.78%	Do not switch
3	DDB dep. = (0.4) (100% - 52%) SL dep. (1/3.5) (100% -52%)	19.20% 13.71%	Do not switch
4	DDB dep. = (0.4) (100%-71.20%) SL dep. (1/2.5) (100% -71.20)	11.52% 11.52%	Switch to SL
5	SL dep. (1/1.5) (100% - 82.72%)	11.52%	
6	½ year SL dep. = (0.5) (11.52%)	5.76%	

In the year 4, SL depreciation is ≥ DDB depreciation and we switch to SL.

Calculate the depreciation amounts from the percentages

Year (n)	MARCS Percentage (%)		Depreciation basis	Depreciation Amount (D_n)
1	20	*	Rs10,000	Rs 2,000
2	32	*	Rs 10,000	Rs 3,200
3	19.20	*	Rs 10,000	Rs 1,920
4	11.52	*	Rs 10,000	Rs 1,152
5	11.52	*	Rs 10,000	Rs 1,152
6	5.76	*	Rs 10,000	Rs 576

7.3 Depreciation Rates applied in Nepal

Class	Assets Included	Rate Applied
A	Buildings, structures, and similar works of a permanent nature	5%
B	Computers, data handling equipment, fixtures, office furniture, and office equipment	25%
C	Automobiles, buses, and minibuses and others	20%
D	Construction and earth moving equipment and any depreciable asset not	15%

	included in another classes	
E	Intangible assets other than depreciable assets included in Class "D"	Rate in percent calculated as divided by the useful life of the asset in the pool.

7.4 Corporate Tax

Corporate tax or company tax refers to a tax imposed on entities that are taxed at the entity level in a particular jurisdiction. Such taxes may include income or other taxes. The tax systems of most countries impose an income tax at the entity level on certain type(s) of entities (company or corporation). Many systems additionally tax owners or members of those entities on dividends or other distributions by the entity to the members. The tax generally is imposed on net taxable income. Net taxable income for corporate tax is generally financial statement income with modifications, and may be defined in great detail within the system. The rate of tax varies by jurisdiction. The tax may have an alternative base, such as assets, payroll, or income computed in an alternative manner.

Most income tax systems provide that certain types of corporate events are not taxable transactions. These generally include events related to formation or reorganization of the corporation. In addition, most systems provide specific rules for taxation of the entity and/or its members upon winding up or dissolution of the entity.

In systems where financing costs are allowed as reductions of the tax base (tax deductions), rules may apply that differentiate between classes of member-provided financing. In such systems, items characterized as interest may be deductible, subject to interest limitations, while items characterized as dividends are not. Some systems limit deductions based on simple formulas, such as a debt-to-equity ratio, while other systems have more complex rules.

Some systems provide a mechanism whereby groups of related corporations may obtain benefit from losses, credits, or other items of all members within the group. Mechanisms include combined or consolidated returns as well as group relief (direct benefit from items of another member).

Most systems also tax company shareholders on distribution of earnings as dividends. A few systems provide for partial integration of entity and member taxation. This is often accomplished by "imputation systems" or franking credits. In the past, mechanisms have existed for advance payment of member tax by corporations, with such payment offsetting entity level tax.

Many systems (particularly sub-country level systems) impose a tax on particular corporate attributes. Such non-income taxes may be based on capital stock issued or authorized (either by number of shares or value), total equity, net capital, or other measures unique to corporations.

Corporations, like other entities, may be subject to withholding tax obligations upon making certain varieties of payments to others. These obligations are generally not the tax of the corporation, but the system may impose penalties on the corporation or its officers or employees for failing to withhold and pay over such taxes.

7.5 After Tax Cash Flow

The after tax cash flow is the net proceeds from an income-producing property, after all costs (taxes, mortgage interest, maintenance costs etc.) of owning and operating the property have been deducted. The amount remaining after expenses, mortgage payments, and income taxes have been deducted from the gross income of an investment property is the after tax cash flow.

Cash Flow Calculation

Net Operating Income less Debt Service less Capital Additions plus Loan Proceeds plus Interest Earned = Cash Flow before Taxes (CFBT)

And,

Cash Flow before Taxes (CFBT) less Income Tax Liability = Cash Flow after Taxes (CFAT)

Now let's look at the calculation to be sure that we understand its various components.

1. Net operating income (NOI) is gross scheduled income less vacancy allowance less operating expenses.
2. Debt service is the total loan payment (first, second, third loans) including principal and interest.
3. Capital additions (different from maintenance and repairs) are improvements to the property having a useful life of more than one year and likely to increase (not merely maintain) the life of the property.
4. Loan proceeds refer to the proceeds obtained from subsequent financing not to the original mortgage, where you might obtain a Rs40, 000 second mortgage to cover the cost of constructing a Rs40, 000 garage for instance.

Example 7.7

Say you have a property with ten tenants each paying Rs1, 000 per month (Rs120, 000 per year). You estimate a vacancy and credit loss of 5%. The property has operating expenses of Rs 45,600 per year, and a first mortgage payment of Rs 36,326 per year. In month six, you add a new roof at the cost of Rs 20,000 and take out Rs 20,000 second mortgage to cover the cost of that construction. Your payment on this loan totals Rs 881 for the remaining six months. What is your property's cash flow before tax (CFBT)?

Solution

(a) Gross Scheduled Income = Rs. 120,000

(b) Vacancy = Rs. 6,000
(c) Gross Schedule Income – Vacancy (a-b) = 120,000 - 6,000
(d) Gross Operating Income = Rs. 114,000
(e) Operating expenses = Rs. 45,600
(f) Gross Operating Income - Operating expenses (d-e) = 114,000 – 45,600
(g) Net Operating income = Rs. 68,400
(h) Cash flow before tax (CFBT) = Net operating income – debt service – capital addition + loan proceed = 68,400 - 36,326 – 20,000 + 20,000 = Rs.32,074

If your tax liability in year one is Rs 7,000, than what is your property's cash flow after taxes (CFAT)?
Cash Flow before Tax 32,074 less Taxes Due 7,000
= Cash Flow after Tax Rs 25,074

Some Solved Examples

1. Some Special handling device can be obtained for $ 12000 is expected to have $600 salvage value at the end of its useful life for four years. Compute depreciation schedule for the devices using i) Straight line method, ii) Sum of year digit method, iii) Double declining balance method and iv) Double declining method to optimal switch to straight line method. Briefly explain corporate tax. (TU,IOE,2069)

Solution
Given: I = $12,000, SV = $ 600, N =4 years

Using Straight line (SL) method
We know, $D_n = (I-S)/N$ = (12000 – 600)/ 4 = $ 2850
Each year $2850 is depreciated.

n	B_{n-1} ($)	D_n ($)	B_n ($)
1	12000	2850	9150
2	9150	2850	6300
3	6300	2850	3450
4	3450	2850	600

Using Sum of year digit (SOYD)

We know, $D_n = \frac{\{N - n + 1\} (I - S)}{SOYD}$

SOYD = 1+2+3+4 = 10 or 4 (4+1) / 2 = 10

End of Year	D_n ($)	B_n ($)
1	4/10 (12,000 – 600) =4,560	12,000 – 4,560 = 7,440
2	3/10 (12,000 – 600) =3,420	7, 440 – 3,420 = 4,020
3	2/10 (12,000 – 600) =2,280	4,008– 2,280= 1,740
4	1/10 (12,000 – 600) =1,140	1,740– 1,140 = 600

Using Double Declining balance

The declining balance rate is 200% of SL i.e. (1/N)*2
= (1/4) *2 = 50%

n	B_{n-1} ($)	D_n ($)=0.5* B_{n-1}	B_n ($)
1	12,000	6,000	6,000
2	6,000	3,000	3,000
3	3,000	1,500	1,500
4	1,500	9,00	6,00

Switching from declining balance to straight line

If switch to SL Beginning of year	SL Depreciation	DDB Depreciation	Decision
1	(12000-600)/4 = 2850	<6000	Do not switch
2	(6000 - 600)/3=1800	<3000	Do not switch
3.	(3000 -600) /2 = 1200	<1500	Do not switch
4	(1500- 600)/ 1 = 900	=900	Switch to SL

The optimal year for switching is year 4

Year	DDB with switching to SL ($)	End of Year Book Value ($)
1	6000	6000
2	3000	3000
3	1500	1500
4	600	600

2. A machine costs Rs. 15000. Its useful life is 5 years and salvage value is Rs 900. Compute the annual depreciation allowances and resulting book value using double declining balance depreciation method.

Solution

For double declining depreciation, we can take the depreciation rate as double the straight line i.e. (1/N)*200% = (1/5) *200= 40%

The depreciation schedule and remaining book value is

n	Book value at beginning of year B_{n-1} (Rs)	Depreciation Charges D_n (Rs) $=0.4^* B_{n-1}$	Remaining Book Value B_n (Rs.)
1	15,000	6,000	9,000
2	9,000	3,600	5,400
3	5,400	2,160	3,240
4	3,240	1,296	1,944
5	1,944	777.6	1,166.4

Here Book value at the end of year 5 is greater than salvage value. We use the switching fundamentals where DDB is switched to SL in any year between 1 to 5 which is optimal year.

SL depreciation for switching is

$$D_n = \frac{\text{book value at the beginning of year n} - \text{salvage value}}{\text{Remaining useful life at beginning of year n}}$$

If switch to SL Beginning of year	SL Depreciation	DDB Depreciation	Decision
1	(15000-900)/5 = 2820	<6000	Do not switch
2	(9000 -900)/4 = 2025	<3600	Do not switch
3	(5400 -900) /3 = 1500	<2160	Do not switch
4	(3240 - 900)/ 2 = 1170	<1296	Do not switch
5	(1944 - 900)/ 1 = 1044	>777.6	Switch to SL

The optimal year for switching is year 4. The depreciation schedule is as follows.

Year	DDB with switching to SL (Rs)	End of Year Book Value (Rs)
1	6000	9000
2	3600	5400
3	2160	3240
4	1296	1944
5	1044	900

STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- To understand the concept of risk, certainty and uncertainty
- To understand the method for dealing with uncertainty (break even analysis, sensitivity analysis, optimistic-pessimistic estimation, risk adjusted MARR and reduction of useful life)
- To understand the concept of Break Even Analysis.
- To understand the concept of decision tree and its application
- To understand the concept of sensitivity analysis and its application.

8.1 Introduction

The decision to make a major capital investment such as introducing a new product requires cash flow information over the life of a project. The profitability estimate of an investment depends on cash flow estimations, which are generally uncertain. The factors to be estimated include the total market for the product; the market share that the firm can attain: the growth in the market; the cost of producing the product, including labor and materials; the selling price; the life of the project; the cost and life of the equipment needed; and the effective tax rates. Many of these factors are subject to substantial uncertainty. A common approach is to make single-number "best estimates" for each of the uncertain factors and then to calculate measures of profitability, such as NPW or rate of return for the project. This approach has two drawbacks:

- No guarantees can ever ensure that the "best estimates" will ever match actual values.
- No provision is made to measure the risk associated with the investment or the project risk. In particular, managers have no way of determining either the

probability that a project will lose money or the probability that it will generate profit.

Because cash flows can be so difficult to estimate accurately, project managers frequently consider a range of possible values for cash flow elements. If a range of values for individual cash flows is possible, it follows that a range of values for the NPW of a given project is also possible. Clearly, the analyst will want to try to gauge the probability and reliability of individual cash flows occurring and, consequently the level of uncertainty about overall project worth.

Quantitative statements about risk are given as numerical probabilities or as values for likelihood of occurrence. Probabilities are given as decimal fractions in the interval 0.0 to 1.0. An event or outcome that is certain to occur has a probability of 1.0. As the probability of an event approaches 0, the event becomes increasingly less likely to occur. The assignment of probabilities of the various outcomes of an investment project is generally called **risk analysis.**

All things in the world vary-one from another, over time, and with different environments. We are guaranteed that variation will occur in engineering economy due to its emphasis on decision making for the future. Except for the use of breakeven analysis, sensitivity analysis, and a very brief introduction to expected values, virtually all our estimates have been certain, i.e. no variation in the amount has entered into the computation of PW, AW, ROR, or any relation used. Certainty is not present in the real world now and surely not in the future. We can observe outcomes with a high degree of certainty, but even this depends upon the accuracy and precision of the scale or measuring instruments. To allow a parameter of an engineering economic study to vary implies that risk, and possible uncertainty is introduced.

Risk

When there may be two or more observable values for a parameter and it is possible to estimate the chance that each value may occur, risk is present.

Uncertainty

Decision making under uncertainty means there are two or more values observable, but the chance of their occurring cannot be estimated or no one is willing to assign the chances. The observable value in uncertainty analysis is often referred to as states of nature.

Decision making Under Certainty

Deterministic estimates are made and entered into measure of worth relations-PW, FW, AW, ROR, B/C –and decision making is based on the results. The values estimated can be considered the most likely to occur with all chance placed on the single value estimate.

Decision making Under Risk

Decision situation where several 'states are possible and probabilities of their occurrence are explicitly stated.

Risk and Uncertainty are different, though complementary concept. Risk refers to a situation where a project has a number of possible alternative outcomes, but the probability of each occurring is known. Uncertainty refers to a situation in which these probabilities are not known. In practice, the terms are often used interchangeably. To insist on the separateness of meaning of these terms would perhaps be slightly legalistic. When we call an investment risky, we mean that we are uncertain about the future outcome of the investment in practice.

8.2 Sources of Uncertainty

It is useful to consider some of the factors that affect the uncertainty involved in the analysis of the future economic consequences of an engineering project. It would be almost impossible to list and discuss all of the potential factors. There are four major sources of uncertainty which are nearly always present in engineering economy studies.

1. Possible inaccuracy of the cash flow estimates used in the study.
2. Type of business involved in relation to the future health of the economy.
3. Type of the physical plant and equipment involved.
4. Length of the study period used in the analysis

8.3 Method of Describing Project Risk

We may begin analyzing project risk by first determining the uncertainty inherent in a project's cash flow. We can do this analysis in a number of ways, which range from making informal judgments to calculating complex economic and statistical analysis. To make a decision on this type, we adopt following methods:

1. **Break even analysis:**
   This is done when selection among alternatives is heavily dependent on a single parameter.
2. **Sensitivity analysis**: This is used when one/more factors are subject to uncertain.
3. **Optimistic- pessimistic – most likely estimation. (Scenario Analysis)**
4. **Risk adjusted MARR**
5. **Reduction of useful life**:

8.4 Break Even Analysis

One of the most common tools used in evaluating the economic feasibility of a new enterprise or product is the

Break-Even Analysis. The main objective of Break-Even Analysis is to find out the condition of no loss and no gain. When one of the engineering economy symbols – P, F, A, I or n – is not known or not estimated, a break even quantity can be determined by setting an equivalence relation for PW or AW equal to zero. This form of break even analysis has been used many times so far. For example, we have solved for the rate of return i*, found for payback period, and determined the P, F, A, or salvage value S at which a series of cash flow estimates return a specific MARR.

Break-Even Analysis for a single project

The break-even point is the point at which revenue is exactly equal to costs. At the breakeven point, no profit is being made nor is any losses incurred. Break-even analysis is based on two types of costs: fixed costs and variable costs.

Fixed cost: These are costs that are the same regardless of how many items you sell. It remains constant over a wide range of activities as long as the business doesn't permanently discontinue operations. All start-up costs, such as rent, insurance and computers, etc are considered fixed costs since you have to make these outlays before you sell your first item.

Variable Cost: These are recurring costs that you absorb with each unit you sell. Variable cost changes with production level, workforce size, and other parameters. It includes costs such as direct labor, materials, indirect costs, advertisements and warranty.

Let,

s be the selling price per unit.

v be the variable cost per unit.

FC is the fixed cost per period.

Q is the quantity of production.

Total sales (S) $= s^*Q$ .... (1)

Total cost of Firm (TC) = Variable cost + Fixed cost

(TC) $= v^*Q + FC$..(2)

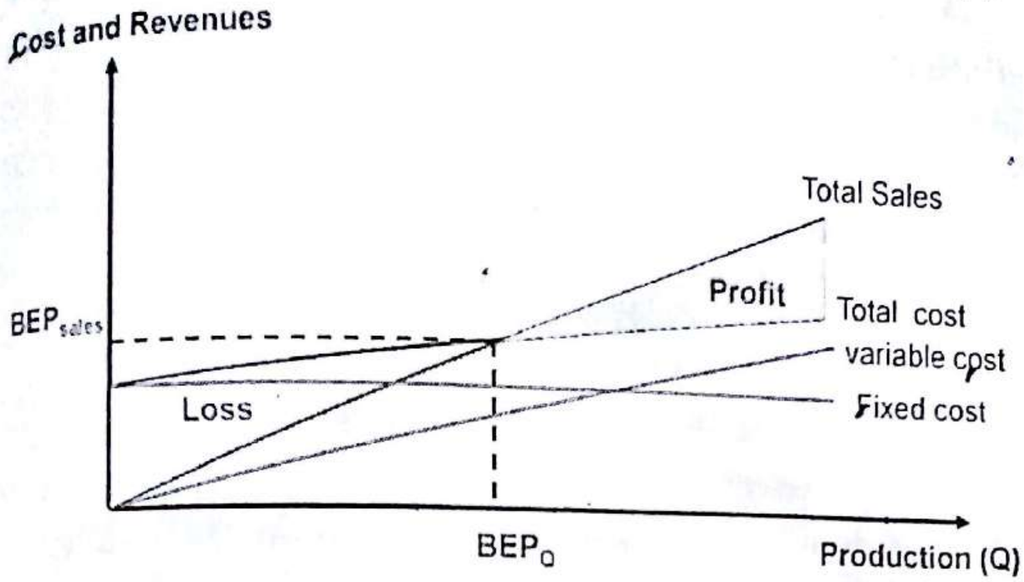


Fig8.1: Break even analysis

Linear plot of these two equations is given in the figure above. The intersection point of the total sales revenues and the total cost line is called the *breakeven point*. At the intersection point the total cost is equal to total revenue and is the condition of no loss or no gain.

$$\text{Total Cost} = \text{Total Sales}$$

$$\text{Variable cost} + \text{fixed cost} = s^*Q$$

$$\text{Or,} \quad v^*Q + FC = s^*Q$$

$$\text{Or,} \quad FC = v^*Q - s^*Q$$

$$\text{Or,} \quad Q = FC / (s - v)$$

$$Q_{BEP} = FC / (s - v) \text{ (units)}$$

Example 8.1

A company produces an electronic timing switch that is used in consumer and commercial products made by several other manufacturing firms. The fixed cost and the total cost are Rs. 40,000 and Rs. 85,000 respectively. The total sales are Rs.105, 000 and sales volume is 15,000 for this situation,

(a) Find breakeven point in terms of number of units.

(b) What should be the output if the profit desired is Rs.50, 000? **(TU,IOE, 2063)**

Solution

Given,

Fixed cost (FC) = Rs 40,000

Total cost (TC) = Rs 85,000

Total sales (S) = Rs 105,000

Sales volume = 15,000

Breakeven point (Q BEP) =?

Breakeven point (Q BEP) =? (If profit = Rs50, 000)

(a) Finding the Breakeven point in terms of number of units

We know,

Total Cost (TC) = Fixed cost (FC) + Variable Cost (VC)

VC = 85,000 – 40,000

VC = Rs.45, 000

Variable cost per unit (v) = 45,000/15,000 = Rs3 per unit

Selling cost per unit (s) = 105,000/15,000 = Rs 7 per unit

Q_{BEP} = FC / (s – v) (units)

Q_{BEP} = 40,000 / (s – v) (units)

Q_{BEP} = 40,000 / (7 – 3) (units)

Q_{BEP} = 10,000 (units) Ans.

(b) If the profit desired is Rs 50,000

Profit = Total Sales – Total Cost

50,000 = s*Q – FC + VC

50,000 = s*Q – (40,000 + v *Q)

90,000 = (7-3)*Q

22,500 = Q

Break even units for the profit to be Rs 50,000 is **22,500 units. (Ans)**

Example 8.2

XYZ Transport Corporation assembles up to 30 trailers per month for 18-wheel trucks in its east coast facility. Production has dropped to 25 units per month over the last 5 months due to a worldwide economic slowdown in transportation service. The following information is available.

Fixed cost (FC) = Rs.7, 50,000 per month

Variable cost per unit (v) = Rs 35,000

Sales per unit (s) = Rs 75,000

(a) How does the reduced production level of 25 units per month compare with the current breakeven point?

(b) What is the current profit level per month for the facility?

(c) What is the difference between the revenue and variable cost per trailer that is necessary to break even at a monthly production level of 15 units, if fixed costs remain constant?

Solution

(a) First develop linear relations for sales (Rs) and the total cost (TC).

Let Q be the quantity in units per month.

Total Sales (S) = s*Q = 75,000Q

Variable Cost (VC) = v*Q = 35,000Q

Total Cost = Fixed cost (FC) + Variable cost (VC)

TC = 750,000 + 35,000Q

For the Breakeven point,

Total Cost	=	Total sales
750,000 + 35,000Q	=	75,000Q
750,000	=	Rs.40, 000 Q
Q_{BEP}	=	18.75 units (19 units

approximately) **(Ans)**

(b) To estimate the profit at Q = 25 units per month,

Profit = Total Sales – Total Cost

= Total sales – (Fixed cost + variable cost)

= (s*Q) – (FC +v*Q)

= (s-v) Q – FC

= (75,000- 35,000)*25 – 7, 50,000

= **Rs. 2, 50,000**

There is a profit of Rs 2, 50,000 per month currently. **(Ans)**

(c) To determine the required difference (s-v) with

Profit = 0, Q = 15 units, and FC = Rs.750, 000

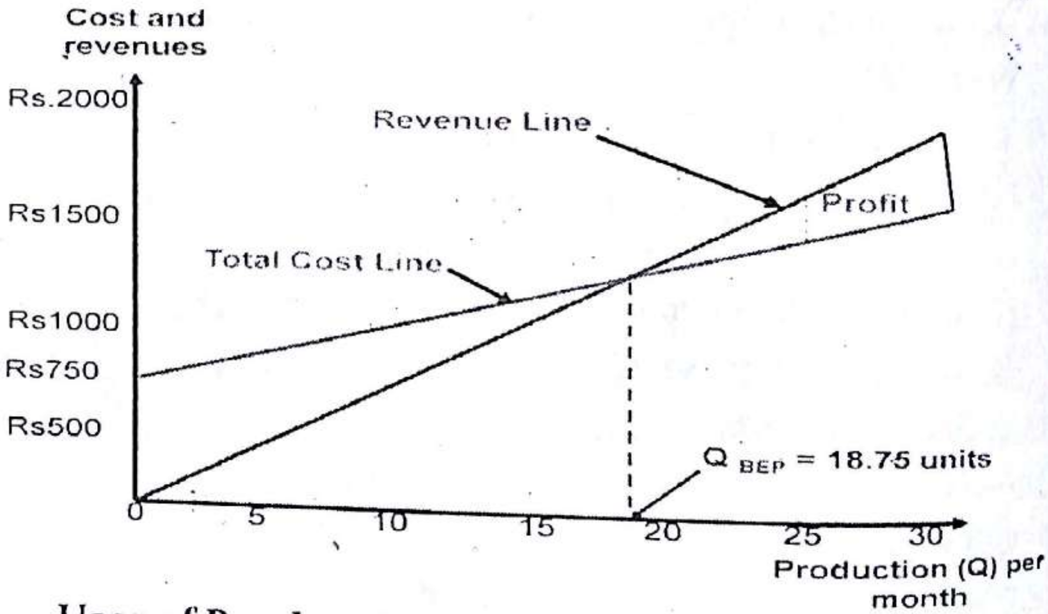
Profit = (s-v) Q – FC

0 = (s-v) 15 - Rs.750, 000

(s - v) = Rs. 750,000/15

(s - v) = Rs. 50,000

If the variable cost per unit (v) remains constant at Rs.35, 000, the sales per unit should be Rs. 85,000 to breakeven at production level of **Q =15 units. (Ans)**



Uses of Break even principle while comparing two alternatives

When the selection between two engineering project alternatives is heavily dependent upon a single factor, we can solve for the value of that factor. That value is known as the breakeven point, i.e. the value at which we are indifferent between the two alternatives. Then, if the best estimate of the actual outcome of the common factor is higher or lower than the breakeven point, and assumed certain, the best alternative become apparent. This break even value calculation is similar to that used to calculate internal rate of return

In mathematical terms,

$$EW_A = f_1(y) \text{ and } EW_B = f_2(y)$$

Where, EW_A = Equivalent worth calculation for the net cash flow of Alternative A

EW_B = Equivalent worth calculation for the net cash flow of Alternative B

y = a common factor affecting the equivalent worth values of Alternative A and Alternative B

The breakeven point between Alternative A and Alternative B is the value of factor y for which the two equivalents worth values are equal. That is $EW_A = EW_B$, or $f_1(y) = f_2(y)$, which may be solved for y.

Examples of common factors

1. **Annual revenues:** solve for the annual revenues to equal (break even) with annual expenses.
2. **Rate of return:** solve for the rate of return on the increment of invested capital at which two given alternatives are equally desirable.
3. **Market (salvage) value:** solve for the future resale value that would result in indifference as to preference for an alternative.
4. **Equipment life:** solve for the useful life required for alternative to be economically justified.
5. **Capacity utilization:** solve for the hours of utilization per year.

Steps to determine breakeven point of common variable

- Define the common variable and its dimensional units.
- Use PW or AW analysis to express the total cost of each alternative as a function of the common variable.

- Equate the two relations and solve for the breakeven value of the variable.
- If the anticipated level is below breakeven value, select alternative with the higher variable cost (larger slope). If the anticipated level is above the break even value select the alternative with lower variable cost (smaller slope).

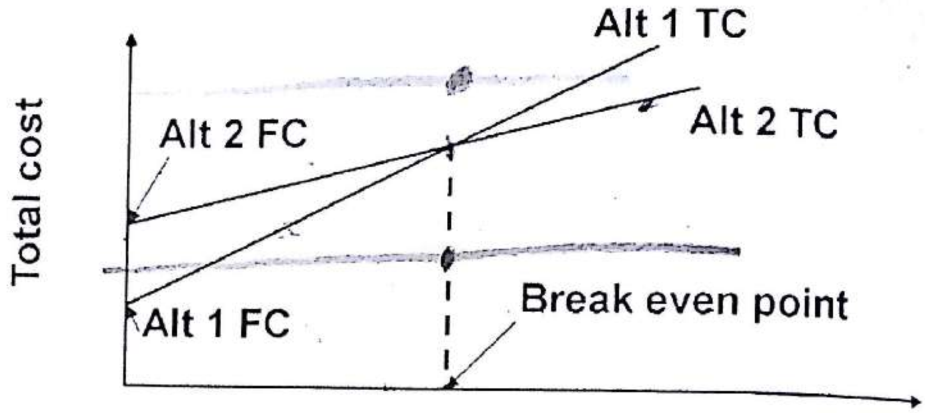


Fig 8.2: Break even analysis of mutually exclusive project

Example 8.3

Suppose that there are two alternative electric motors that provide 100-hp output.

Item	Alpha motor	Beta motor
Purchase cost	Rs 12,500	Rs 16,000
Efficiency	74%	92%
Maintenance Cost	Rs500 per year	Rs250 per year
Life	10 years	10 years
Annual tax& insurance	1.5% of the investment	
MARR	15%	

(a) How many hours per year would the motors have to be operated at full load for the annual cost to be equal? Electricity cost = Rs 0.05 per KW hour.

(b) If annual operation hour is 600 hrs, which motor should be selected?

Solution

(a) Alpha Motor

Calculating the Annual equivalent cost.

1. Capital Recovery (CR)

   = Rs.12, 500 (A/P, 15%, 10) – 0

   = Rs. 2,490 per year

2. Maintenance Cost

   = Rs. 500 per year

3. Tax and insurance

   = 1.5% of initial investment

   = 1.5% of 12,500 = Rs. 187 per year

4. Operating expenses for power (electricity cost)

   Let 'X' be the number of hours of operation per year

   = {(0.05) * (100)*0.746) * X}/ 0.74

   = Rs. 5.04 X per year

Total annual equivalent (AW) cost of Alpha motor

   = Rs. 2,490 + Rs 500 + Rs. 187 + Rs. 5.04 X

   = Rs. 3177 + 5.04 X ....(i)

(b) Beta Motor

Calculating the Annual equivalent cost.

1. Capital Recovery (CR)

   = Rs. 16,000 (A/P, 15%, 10) – 0

   = Rs. 3,190 per year

2. Maintenance Cost

   = Rs. 250 per year

3. Tax and insurance

   = 1.5% of initial investment

   = 1.5% of 16,000 = Rs. 240 per year

4. Operating expenses for power (electricity cost)

   Let 'X' be the number of hours of operation per year

   = {(0.05) * (100)*0.746) * X} / 0.92

   = Rs. 4.05 X per year

Total annual equivalent (AW) cost of Beta motor

   = Rs. 3,190 + Rs. 250 + Rs. 240 + Rs. 4.05 X

$= Rs.\ 3680 + 4.05\ X$..........................(ii)

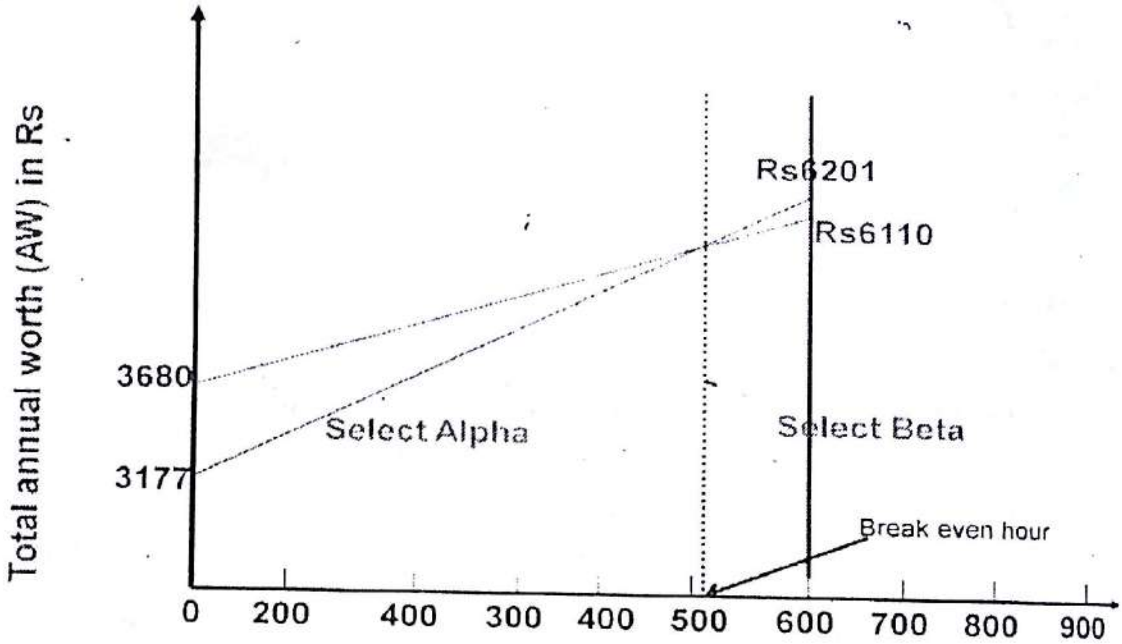
At the Breakeven point,

	AW_{alpha}	=	AW_{beta}
Or,	$Rs.3177 + Rs.5.04\ X$	=	$Rs.3,680 + Rs.4.05X$
Or,	$Rs.5.04\ X - 4.05\ X$	=	$Rs.3,680 - Rs.3,177$
Or,	$X = Rs.503/0.99 = 508$ hours/year		

(b) If the operation hours is 600hrs per year,

$AW_{alpha} = Rs.\ 3,177 + Rs.\ 5.04 * 600 = Rs.6,201$

$AW_{beta} = Rs.3680 + Rs.\ 4.05 * 600 = Rs.6,110$



Example 8.4

CFL bulb of 20 watt costs Rs 250 where as Filament bulb of 100 watt (20 watt CFL bulb is equivalent to 100 watt Filament bulb) costs Rs 30. Which bulb do you prefer in your house? Why? Electricity cost is Rs 10 per unit (Kw-hour)

Solution

CFL bulb

1. Initial cost = Rs 250
2. Cost of use = $(20/1000)*x*10 = 0.2x$ (where x is the number of hours lighting the bulb)
3. Total cost = Rs $250 + 0.2x$

Filament bulb

1. Initial cost = Rs 30
2. Cost of use = (100/1000)*x*10 = x (where x is the number of hours lighting the bulb)
3. Total cost = Rs.30 + x

At Breakeven

Total cost of CFL bulb = Total cost of Filament bulb

Rs 250 + 0.2x = Rs.30 + x

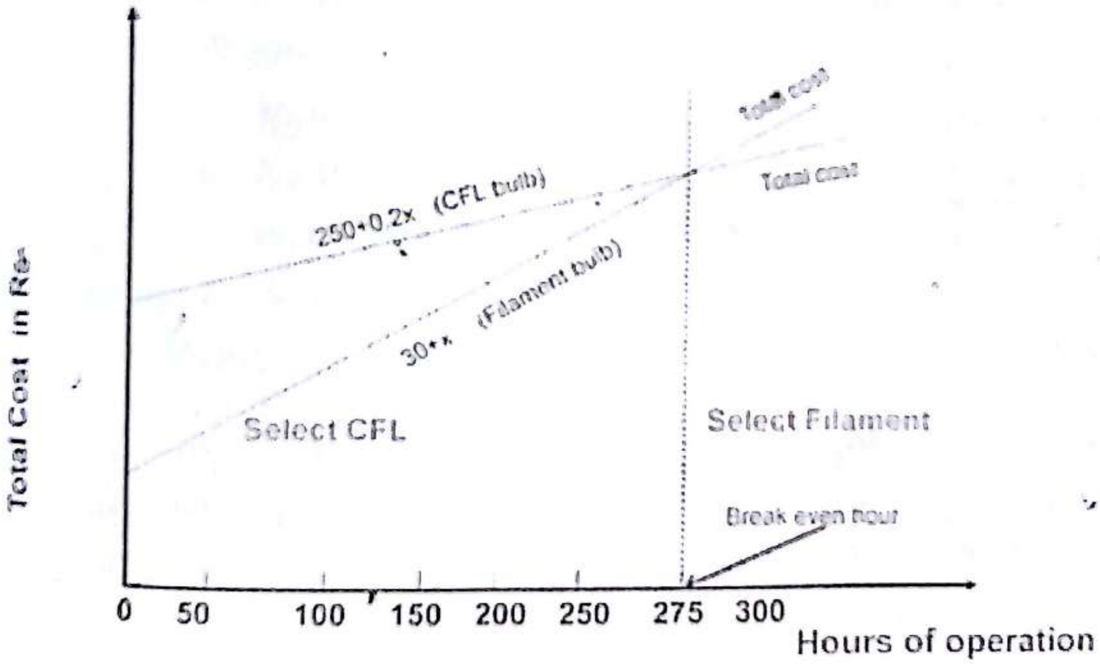
Rs 220 = 0.8x

x = 275 hours

In a day, we light the bulb for approximately 4 hours

Therefore, x = 275/4 = 69 days

Since I have to use for more than 275 hours (69days), I will use CFL bulb in my house.



8.5 Sensitivity Analysis

One way to glean (collect) a sense of the possible outcomes of an investment is to perform a sensitivity analysis. Sensitivity analysis determines the effect on the NPW of variations in the input variables (such as revenues, operating, cost, salvage value, useful life etc) used to estimate after tax cash flows. A **Sensitivity** analysis reveals how much the NPW (or some other criteria of merit such as NFW, NAW, NFW, IRR, and

BCR) will change in response to a given change in an input variable. In a calculation of cash flows, some items have a greater influence on the final result than others. In some problems, the most significant item may be easily identified. For example, the estimate of sales volume is often a major factor in a problem in which the quantity sold varies among the alternatives.

Sensitivity analysis is sometimes called "what if" analysis because it answers questions such as, What if incremental sales are only 1000 units rather than 2000 units? Then what will the NPW be? Sensitivity analysis begins with a base case situation, which is developed using the most likely values for each input. We then change the specific variables of interest by several specified percentage above and below the most likely value, while holding other variables constant. Next, we calculate a new NPW for each of these values. A convenient and useful way to present the results of a sensitivity analysis is to plot **sensitivity graph.** The slopes of the lines show how sensitive the NPW is to changes in each of the inputs. The steeper the slope, the more sensitive the NPW is to change in a particular variable. Sensitivity graph identifies the crucial variables that affect the final outcome most.

"In engineering economic studies, sensitivity analysis is a general non probabilistic methodology, to provide information about the potential impact of uncertainty in selected factor estimates".

Steps for sensitivity analysis

- It begins with the **base case situation**, which is developed using the most likely values for each input.
- **Change the specific variable** of interest by several **specified percentages** above and below the most - likely value, while **holding other variables constant.**

- Calculate a new **PW/FW/AW/IRR/BCR** for each of these values.
- Present the results of a sensitivity analysis in the **sensitivity graph.**
- The slope of the line shows how sensitive the NPW is to changes in each of the inputs.
- The steeper the slope, the more the sensitive the NPW is to change in a particular variable

Interpretation of Sensitivity Graph

- On a plot, there are two directions to measure uncertainty:
    - On X-axis, the uncertainty in the input variable is measured.
    - On Y-axis, the impact of that uncertainty on PW is measured.
- The slope of the line shows how sensitive the PW is to change in each of the inputs. The steeper the slope, the more sensitive the PW is to a change in a particular variable.
- The graph allows us to identify the crucial variables that most affect the final outcome
- A properly drawn spider plot shows the following:
    - Limits of uncertainty for each cash flow element.
    - Impact of each cash flow element on the PW or IRR (each can be used as Y-axis)
    - Identification of each cash flow element that might change the recommendation.

Example 8.5

The best (most likely) cash flow estimates are given below for a new piece of equipment being considered for immediate installation. Because of the new technology built into this machine, it is desired to investigate its PW over a range of

±40% (The estimates varies with the increment of 10% up to 40%) changes in the estimates for

1. Capital Investment (I)
2. Annual net Cash flow (A)
3. Salvage value (SV)
4. Useful life (N)

Based on these best estimates, plot a diagram that summarizes the sensitivity of present worth to percent deviation changes in each separate factor estimate when MARR =10% per year

Capital Investment (I)	- Rs 11,500
Revenues / yr	Rs 5,000
Expenses / yr	- Rs 2,000
Market Value (MV)	Rs1,000
Useful Life (N)	6 years

Solution

1. Developing base case situation i.e. The PW of the project (installation of new equipment) based on the best estimate of the given factors is

   Prime Equation

   PW (10%) = - Rs. 11,500 + Rs.3, 000 (P/A, 10%, 6) + Rs. 1,000 (P/F, 10%, 6)

   = **Rs.2, 130**

2. Changing one of the specific variable (I, A, SV, N) by ±40% of the most – likely value, while holding other variables constant

(a) **When the capital Investment (I) varies with the increment of 10% up to ±40%)**

PW (10%) = - Rs 11,500 (1 ±p %) + Rs.3, 000 (P/A, 10%, 6) + Rs.1, 000 (P/F, 10%, 6)

PW (10%) at + 10% = - Rs 11,500 (1+0.1) + Rs.3, 000 (P/A, 10%, 6) + Rs.1, 000 (P/F, 10%, 6)

= -11,500 *0.1 -Rs 11,500 +Rs.3, 000 (P/A, 10%, 6) + Rs.1, 000 (P/F, 10%, 6)

= +Rs 980

at + 20% = -11,500 *0.2 +2,130 = -Rs. 170

at + 30% = -11,500 *0.3 +2,130 = -Rs. 1,320

at + 40% = -11,500 *0.4+2,130 = -Rs. 2,470

PW (10%) at - 10% = - Rs 11,500 (1-0.1)) + Rs.3, 000 (P/A, 10%, 6) + Rs.1, 000 (P/F, 10%, 6)

= + 11,500 *0.1 -11,500 +Rs.3, 000 (P/A, 10%, 6) + Rs.1, 000 (P/F, 10%, 6)

= +Rs 1,150 +Rs. 2,130 = Rs. +3,280

at - 20% = 0.2* 11,500+ 2,130 = Rs. 4,430

at - 30% = 0.3* 11,500+ 2,130 = Rs. 5,580

at - 40% = 0.4* 11,500+ 2130 = Rs. 6,730

(b) When the Annual Cash Flow (A) varies with the increment of 10% up to ±40%)

PW (10%) at + 10% = - Rs 11,500 + Rs 3,000 (1±p %) (P/A, 10%, 6) + Rs 1,000 (P/F, 10%, 6)

PW (10%) at + 10% = - 11,500 + 3,000 (P/A, 10%, 6) + 3,000 *0.1 (P/A, 10%, 6) + 1,000 (P/F, 10%, 6)

= Rs 2,130 + 0.1* 13,065.8 = Rs. 3,436.58

at + 20% = Rs 2,130 + 0.2 *13,065.8 = Rs. 4,743

at + 30% = Rs 2,130 +0.3 *13,065.8 = Rs. 6,049.74

at + 40% = Rs 2,130 + 0.4 *13,065.8 = Rs. 7,356

PW (10%) at -10% = Rs. 823.42

at -20% = -Rs.483.16

at -30% = -Rs.1,789.74

at -40% = - Rs. 3,096.32

(c) When the Salvage Value (SV) varies with the increment of 10% up to ±40%

PW (10%) at + 10% = - Rs 11,500 + Rs 3,000 (P/A, 10%, 6) + Rs 1,000 (1±p %) (P/F, 10%, 6)

PW (10%) at + 10% = -11,500 + 3,000 (P/A, 10%, 6) + 1,000 (P/F, 10%, 6) + 1000* 0.1 (P/F, 10%, 6) = Rs. 2,186.44

at + 20% = Rs. 2,243

at + 30% = Rs. 2,299

at + 40% = Rs. 2,356

PW (10%) at - 10% = Rs. 2,073.56

at - 20% = Rs. 2,017.12

at - 30% = Rs. 1,960.68

at - 40% = Rs. 1,904.24

(d) When the useful life (N) varies with the increment of 10% up to ±40%

By + 10%, N becomes 6*1.1 =6.6,

PW (10%) = -11,500 + 3,000 (P/A, 10%, 6.6) + 1,000 (P/F, 10%, 6.6) = Rs. 3,040

By + 20%, N becomes 6*1.2 = 7.2

PW (10%) = -11,500 + 3,000 (P/A, 10%, 7.2) + 1,000 (P/F, 10%, 7.2) = Rs. 3,900

By + 30%, N becomes 6*1.3 = 7.8

PW (10%) = -11,500 + 3,000 (P/A, 10%, 7.8) + 1,000 (P/F, 10%, 7.8) = Rs. 4,710

By + 40%, N becomes 6*1.4 = 8.4

PW (10%) = - 11,500 + 3,000 (P/A, 10%, 8.4) + 1,000 (P/F, 10%, 8.4) = Rs. 5,477

By - 10%, N becomes 6*0.9 =5.4,

PW (10%) = -11,500 + 3,000 (P/A, 10%, 5.4) + 1,000 (P/F, 10%, 6.6) = Rs. 1,167

By -20%, N becomes 6*0.8 =4.8,

PW (10%) = -11,500 + 3,000 (P/A, 10%, 4.8) + 1,000 (P/F, 10%, 4.8) = Rs. 147

By -30%, N becomes 6*0.7 =4.2,

PW (10%) = -11,500 + 3,000 (P/A, 10%, 4.2) + 1,000 (P/F, 10%, 4.2) = -Rs.933

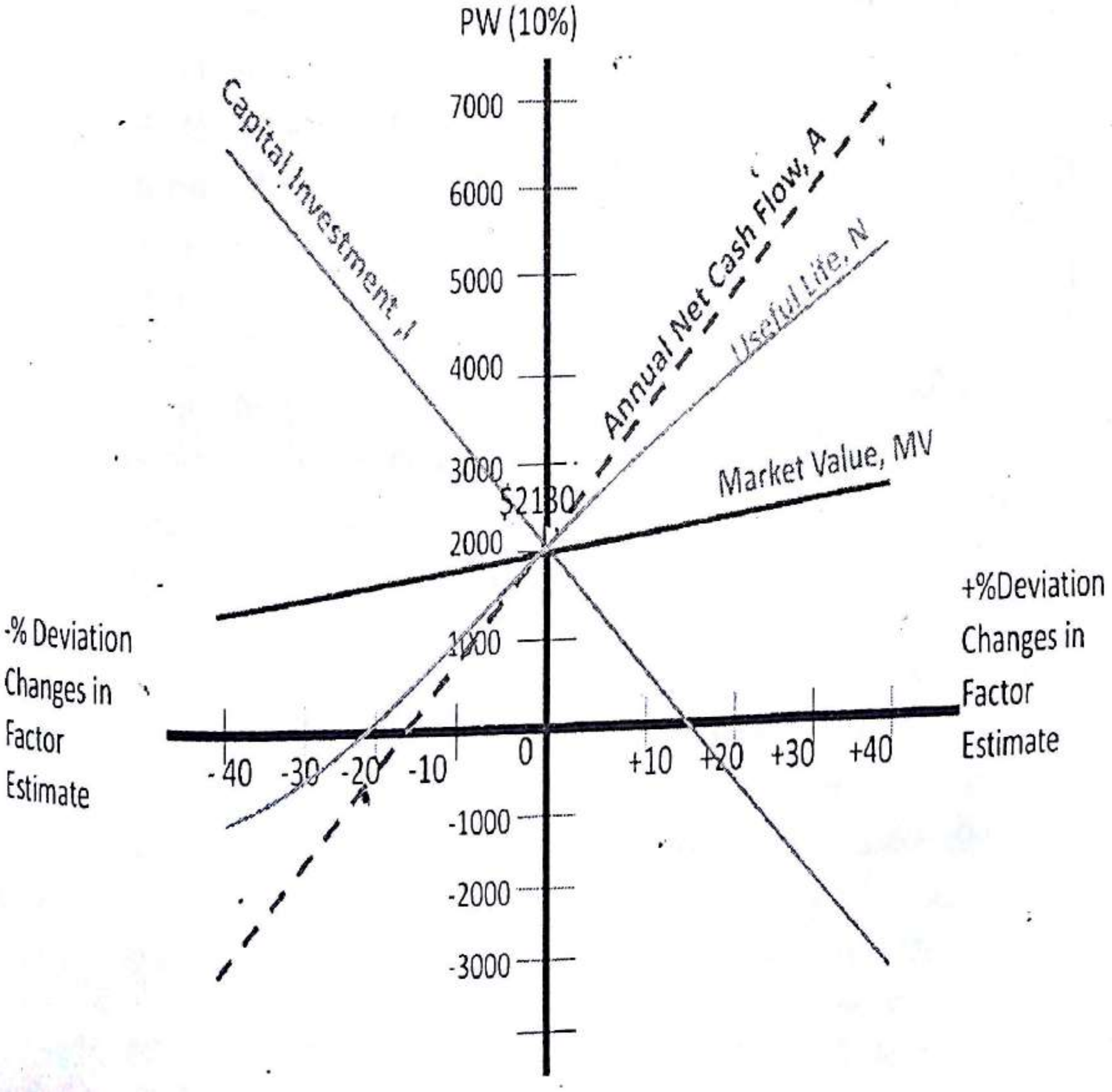
By -40%, N becomes 6*0.7 =3.6,

PW (10%) = -11,500 + 3,000 (P/A, 10%, 3.6) + 1,000 (P/F, 10%, 3.6) = -Rs.2, 786

Present Worth (PW)

	-40 %	-30 %	-20 %	-10 %	0 %	+10 %	+20 %	+30 %	+40 %
(I) (Rs)	6730	5,580	4430	3,280	2,130	980	-170	-1320	-2,470
(A) (Rs)	- 3,096	-1,789	-483	823	2,130	3,436	4,743	6,049	7356
(SV) (Rs)	1,904	1,960.	2,017	2,073	2,130	2,186	2,243	2,299	2,356
(N) (Rs)	- 2,786	-933	147	1167	2,130	3040	3,900	4,710	5,477

3. Plotting these values in the sensitivity graph



4. Revelation of spider plot

- Figure shows the sensitivity of the present worth to percent deviation changes in each factor's best estimate.
- Other factors are assumed to remain at their best estimate values
- The relative degree of sensitivity of the present worth to each factor is indicated by the slope of the curves (the "steeper" the slope of a curve the more sensitive the present worth is to the factor)
- Present worth is insensitive to MV
- Present worth is sensitive to I, A, and N
- Among I, A and N, Investment (I) is more sensitive.

Example 8.6

Perform sensitivity analysis using IRR and BCR (with increment of 10%) over a range of ±30% in (a) Initial investment (b) Net annual revenue (c) Salvage value and (d) Useful life

Initial Investment (I) = Rs. 2, 00,000
Annual Revenues (R) = Rs. 50,000
Annual Expense (E) = Rs. 5,000
Salvage value (SV) = Rs. 25, 000
Useful life (N) = 10 years
MARR =12%per year

Solution

Criteria of Merit: IRR

Using AW formulation, Prime equation becomes

$-2,00,000\ (A/P, i^*\%, 10) + (50,000 - 5,000) + 25,000\ (A/F, i^*\%, 10) = 0$

Or, $-2,00,000\ ((1+i^*)^{10} * i)/((1+i^*)^{10} - 1) + 45,000 + 25,000\ (i^*/((1+i^*)^{10} - 1)) = 0$

IRR = 18.95 %

(a) When the capital investment (I) varies by±30% (with an increment of 10%)

At 10%, AW = -2, 00,000 *1.1 * ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) +45,000 + 25,000 (i*/(1+i*)^{10} -1) = 0

IRR = 16.37%

At 20%, AW = -2, 00,000 *1.2 * ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) +45,000 + 25,000 (i*/(1+i*)^{10} -1) = 0

IRR = 14.15%

At 30%, AW = -2, 00,000 *1.3 * ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) +45,000 + 25,000 (i*/(1+i*)^{10} -1) = 0

IRR = 12.20%

At -10%, AW = -2, 00,000 *0.9 * ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) +45,000 + 25,000 (i*/(1+i*)^{10} -1) =0

IRR = 21.99%

At -20%, AW = -2, 00,000 *0.8 * ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) +45,000 + 25,000 (i*/(1+i*)^{10} -1) =0

IRR = 25.67%

At -30%, AW = -2, 00,000 *0.7 * ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) +45,000 + 25,000 (i*/(1+i*)^{10} -1) =0

IRR = 30.23%

(b) When the net annual revenue (R-E) varies by±30% (with an increment of 10%)

At 10%, AW = -2, 00,000 ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) + 45,000 (1.1) + 25,000 (i/(1+i)^{10} -1) = 0

IRR = 20.66 %

At 20%, AW = -2, 00,000 ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) + 45,000 (1.2) + 25,000 (i*/(1+i*)^{10} -1) = 0

IRR = 23.36 %

At 30%, AW = -2, 00,000 ((1+i*) ^{10} *i)/ (1+i*) ^{10} -1) + 45,000 (1.3) + 25,000 (i*/(1+i*)^{10} -1) = 0

IRR = 26%

At -10%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 (0.9) + 25,000 $(i^*/(1+i^*)^{10} -1) = 0$

IRR = 16.18 %

At -20%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 (0.8) + 25,000 $(i^*/(1+i^*)^{10} -1) = 0$

IRR = 13.32%

At -30%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 (0.7) + 25,000 $(i^*/(1+i^*)^{10} -1) = 0$

IRR =10.35 %

(c) When the salvage value (SV) varies by ±30% (with an increment of 10%)

At 10%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 + 25,000 (1.1) $(i^*/(1+i^*)^{10} -1) = 0$

IRR = 19.01 %

At 20%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 + 25,000 (1.2) $(i^*/(1+i^*)^{10} -1) = 0$

IRR = 19.07 %

At 30%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 + 25,000 (1.3) $(i^*/(1+i^*)^{10} -1) = 0$

IRR = 19.13 %

At -10%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 + 25,000(0.9) $(i^*/(1+i^*)^{10} -1) = 0$

IRR =18.88%

At -20%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 + 25,000(0.8) $(i^*/(1+i^*)^{10} -1) = 0$

IRR = 18.82%

At -30%, AW = -2, 00,000 $((1+i^*)^{10} *i)/ (1+i^*)^{10} -1)$ + 45,000 + 25,000(0.7) $(i^*/(1+i^*)^{10} -1) = 0$

IRR = 18.76%

(d) When the useful life (N) varies by ±30% (with an increment of 10%)

At 10%, N = 10*1.1 = 11, AW = -2, 00,000 $((1+i^*)^{11} *i)/ (1+i^*)^{11} -1)$ + 45,000 + 25,000 $(i/ (1+i)^{11} -1) = 0$

IRR = 19.25 %

At 20%, N = 10*1.2= 12, AW = -2, 00,000 $((1+i^*)^{12} *i)/ (1+i^*)^{12} -1)$ + 45,000 + 25,000 $(i/ (1+i)^{12} -1) = 0$

IRR = 19.96 %

At 30%, N = 10*1.3= 13, AW = -2, 00,000 $((1+i^*)^{13} *i)/ (1+i^*)^{13} -1)$ + 45,000 + 25,000 $(i/ (1+i)^{13} -1) = 0$

IRR = 20.50 %

At -10%, N = 10*0.9 = 9, AW = -2, 00,000 $((1+i^*)^{9} *i)/ (1+i^*)^{9} -1)$ + 45,000 + 25,000 $(i/ (1+i^*)^{9} -1) = 0$

IRR =17.04 %

At -20%, N = 10*0.8 = 8, AW = -2, 00,000 $((1+i^*)^{8}*i)/ (1+i^*)^{8} -1)$ + 45,000 + 25,000 $(i/(1+i^*)^{8}-1) = 0$

IRR = 15.29 %

At -30%, N = 10*0.7 = 7, AW = -2, 00,000 $((1+i^*)^{7} *i)/ (1+i^*)^{7}-1)$ + 45,000 + 25,000 $(i/(1+i^*)^{7} -1) = 0$

IRR = 12.84 %

If we put the values in tabular form

	IRR (%)						
	-30%	-20%	-10%	0%	+10%	+ 20%	+30%
Capital Investment (I) (Rs)	30.23	25.67	21.99	18.95	16.37	14.15	12.20
Annual cash flow (A) (Rs)	10.35	13.32	16.18	18.95	20.66	23.36	26
Salvage Value(SV	18.76	18.82	18.88	18.95	19.01	19.07	19.13

) (Rs)							
Useful Life (N) (Rs)	12.84	15.29	17.04	18.95	19.25	19.96	20.50

Criteria of Merit: Benefit Cost Ratio (BCR)

Using FW formulation, the prime equation to calculate BCR

$BCR_{modified}$ = {(50,000 -5,000) (F/A, 12%, 10)} / 2, 00,000 (F/P, 12%, 10) -25,000

= 7, 89691.5 / 6, 21,700 – 25,000 = 1.32

(a) **When the capital investment (I) varies by±30% (with an increment of 10%)**

At 10%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000*1.1) (F/P, 12%, 10) -25,000

= 7, 89,691.5/ 683870 -25,000 = 1.19

At 20%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000*1.2) (F/P, 12%, 10) -25,000

= 7, 89,691.5/ 7. 45.392 -25,000= 1.09

At 30%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000*1.3) (F/P, 12%, 10) -25,000

= 7, 89,691.5/ 8, 07,508 -25,000 = 1.00

At -10%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000*0.9) (F/P, 12%, 10) -25,000

= 7, 89,691.5/ 5, 59,044 -25,000 = 1.47

At -20%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000*0.8) (F/P, 12%, 10) -25,000

= 7, 89,691.5/ 4, 96,928 -25,000= 1.67

At -30%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000*0.7) (F/P, 12%, 10) -25,000

= 7, 89,691.5/ 4, 34,812 -25,000 = 1.92

(b) **When the capital investment (I) varies by±30% (with an increment of 10%)**

At 10%, BCR = {(50,000 -5,000) (1.1) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 8, 68,660.65 / 6, 21,700 – 25,000 = 1.45

At 20%, BCR = {(50,000 -5,000) (1.2) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 9, 47,629.8 / 6, 21,700 – 25,000 = 1.58

At 30%, BCR = {(50,000 -5,000) (1.3) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 10, 26,598.95 / 6, 21,700 – 25,000 = 1.72

At -10%, BCR = {(50,000 -5,000) (0.9) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 710722.35 / 6, 21,700 – 25,000 = 1.19

At -20%, BCR = {(50,000 -5,000) (0.8) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 6, 31,753.2 / 6, 21,700 – 25,000 = 1.05

At -30%, BCR = {(50,000 -5,000) (0.7) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 5, 52,784.05 / 6, 21,700 – 25,000 = 0.92

(c) When the salvage value (SV) varies by±30% (with an increment of 10%)

At 10%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 7, 89,691.5 / 6, 21,700 – 25,000 (1.1) = 1.32

At 20%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 7, 89691.5 / 6, 21,700 – 25,000 (1.2) = 1.33

At 30%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 7, 89,691.5 / 6, 21,700 – 25,000 (1.3)

= 1.34

At -10%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 7, 89,691.5 / 6, 21,700 – 25,000 (0.9)

= 1.31

At -20%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 7, 89,691.5 / 6, 21,700 – 25,000 (0.8) = 1.31

At -30%, BCR = {(50,000 -5,000) (F/A, 12%, 10)} / (2, 00,000) (F/P, 12%, 10) -25,000

= 7, 89,691.5 / 6, 21,700 – 25,000 (0.7) = 1.30

(d) When the Useful life (N) varies by±30% (with an increment of 10%)

At 10%, N =10*1.1=11, BCR = {(50,000 -5,000) (F/A, 12%, 11)} / 2, 00,000 (F/P, 12%, 11) -25,00

= 9, 29,457 / 6, 95,700 – 25,000 = 1.38

At 20%, N =10*1.2=12, BCR = {(50,000 -5,000) (F/A, 12%, 12)} / 2, 00,000 (F/P, 12%, 12) -25,000

= 10, 85,989.5 / 6, 95,700 – 25,000 = 1.43

At 30%, N =10*1.3=13, BCR = {(50,000 -5,000) (F/A, 12%, 13)} / 2, 00,000 (F/P, 12%, 13) -25,000

= 12, 61,309.5 / 87, 2,700 – 25,000 = 1.48

At -10%, N =10*0.9=9, BCR = {(50,000 -5,000) (F/A, 12%, 9)} / 2, 00,000 (F/P, 12%, 9) -25,000

= 6, 64,906.5 / 5, 54,620 – 25,000 = 1.25

At -20%, N =10*0.8=8, BCR = {(50,000 -5,000) (F/A, 12%, 8)} / 2, 00,000 (F/P, 12%, 8) -25,000

= 5, 53,486.5 / 4, 95,200 – 25,000 = 1.17

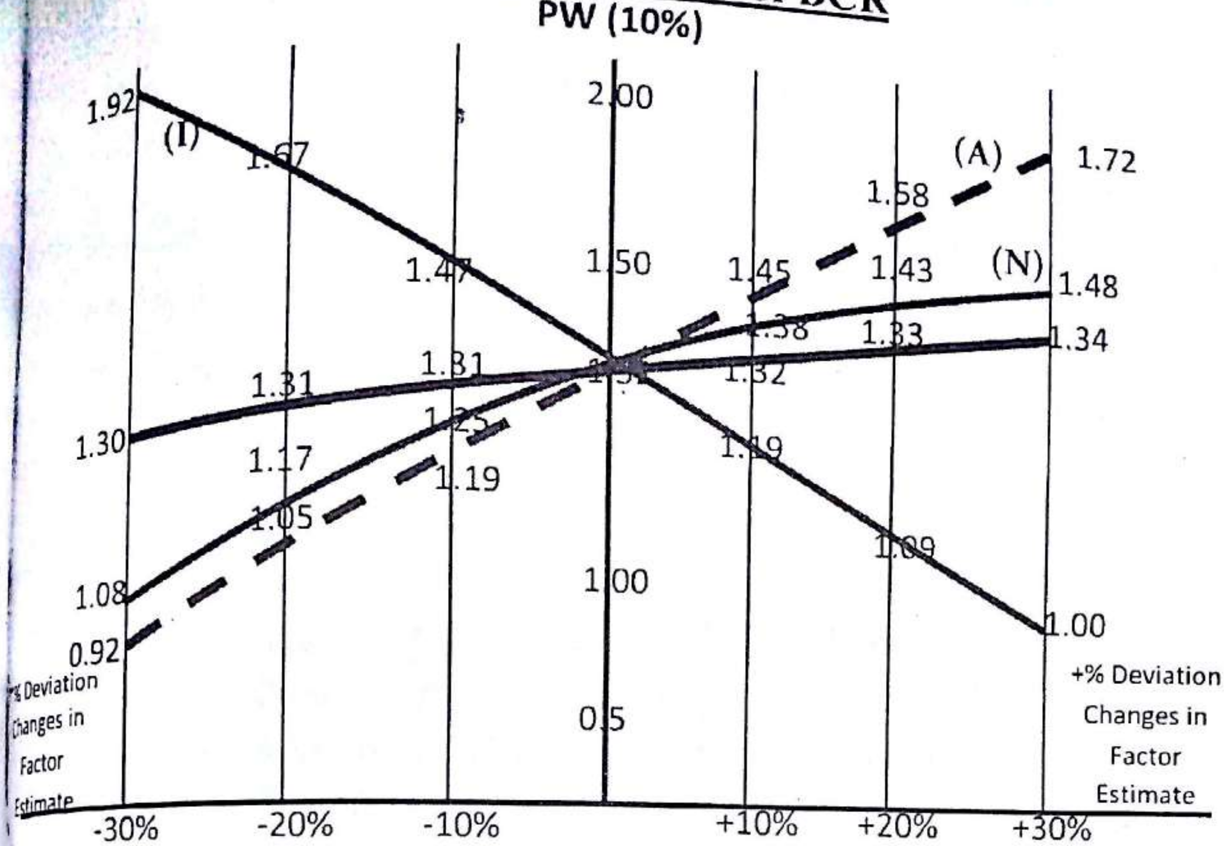
At -30%, N =10*0.7=7, BCR = {(50,000 -5,000) (F/A, 12%, 7)} / 2, 00,000 (F/P, 12%, 7) -25,000

= 4, 54,005 / 4, 42,140 – 25,000 = 1.08

If we put the values in tabular form

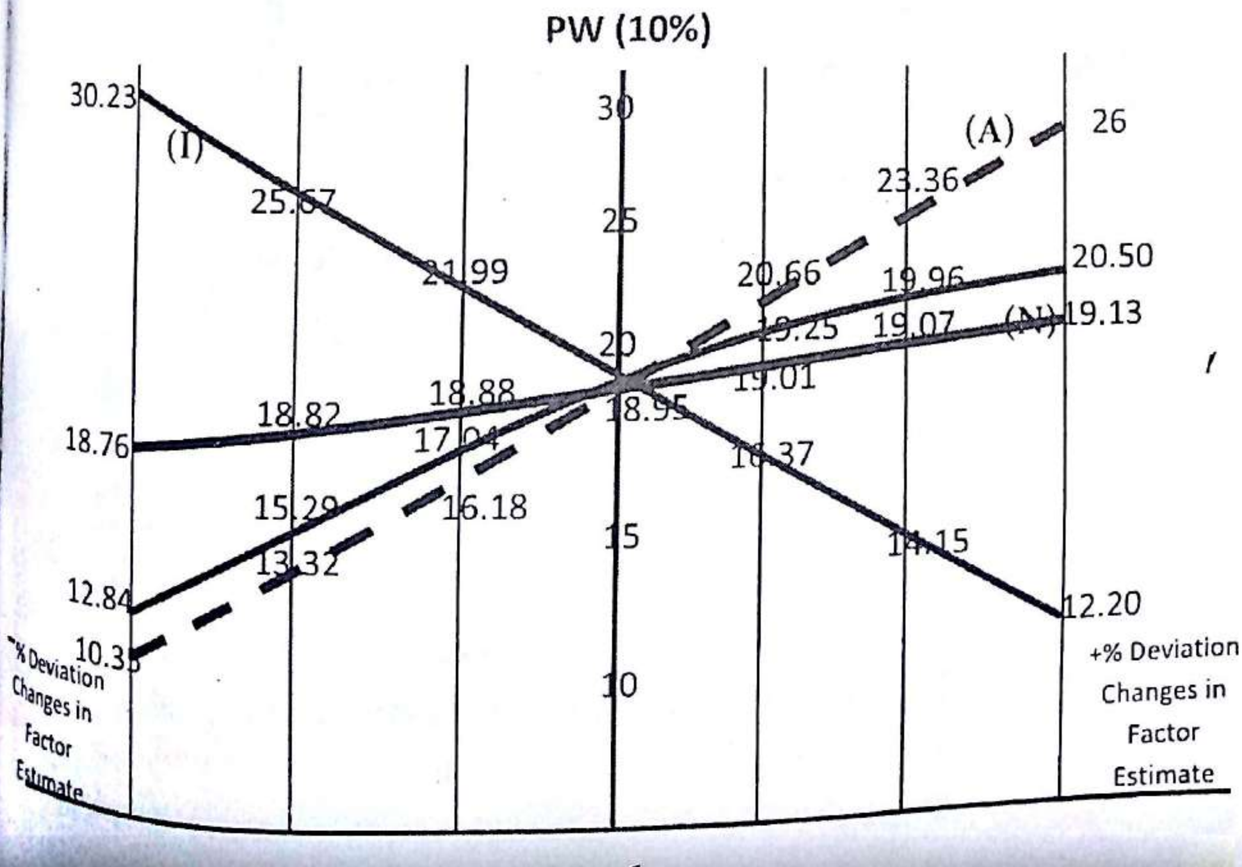
	BCR						
	-30%	-20%	-10%	0%	+ 10%	+ 20%	+ 30%
Capital Investment (I) (Rs)	1.92	1.67	1.47	1.32	1.19	1.09	1.00
Annual cash flow (A) (Rs)	0.92	1.05	1.19	1.32	1.45	1.58	1.72
Salvage Value(SV) (Rs)	1.30	1.31	1.31	1.32	1.32	1.33	1.34
Useful Life (N) (Rs)	1.08	1.17	1.25	1.32	1.38	1.43	1.48

Sensitivity Graph: Merit of BCR



Here, I and A seems to be sensitive. Among I and A, Investment (I) is more sensitive

Sensitivity Graph: Merit of IRR



Here, I and A seems to be sensitive. Among I and A, Investment (I) is more sensitive

Example 6.7

ABC Company limited proposes to start a new venture of the manufacture of fluorescent bulbs. The estimates of the new venture are as under.

Output of bulbs per annum	: 3. 00,000 nos.
Expected sales revenues per annum	: Rs. 1, 50, 00,000
Fixed costs	: Rs. 35, 00,000
Variable costs	: Rs. 66, 00,000

(a) If the selling price comes down to Rs 40.00 per unit, find out its effect on Break Even Point (BEP)

(b) If the fixed cost increase to Rs. 40,00,000, find out its effect on Break Even Point (BEP)

(c) If the variable cost increases by 10%, find out its effect on Break Even Point (BEP)

Solution

Fixed Cost = Rs. 35, 00,000

Sales Revenue = Rs. 1, 50, 00,000

Selling price per unit (1, 50, 00,000/3, 00,000) = Rs. 50

Variable cost per unit (66, 00,000/3, 00,000) = Rs. 22

Break Even Point (BEP)

= Fixed Cost/ (selling price per unit – variable cost per unit)

= 35, 00,000/ (50-22)

= 1, 25,000 units

(a) If the selling price per unit comes down to Rs 40.00
BEP = 35, 00,000 / (40-22) = 1, 94,444 units

(b) If cost increases to Rs. 40,00,000
BEP = 40, 00,000/ (50-22) = 1, 42,875 units

(c) If variable cost increases by 10%

Revise variable cost per unit = (1.1*22) = Rs. 24.20

BEP = 35, 00,000/ (50-24.20) = 1, 35, 659 units

Results:

(a)

Selling Price per unit (Rs.)	BEP	Reduction in (selling Price)	Increase in BEP
50 40	1,25,000 units 1,94,444units	(50-40)*100 / 50 = 20%	(1,94,444 -1,25,000)*100/ 1,25,000 = 56%

(b)

Fixed Cost (Rs.)	BEP	Increase in fixed cost	Increase in BEP
35,00,000 40,00,000	1,25,000 units 1,42,875units	(40,00,000 – 35,00,000)*100 / 35,00,000 = 14.21%	(1,42,875 - 1,25,000)*100/ 1,25,000 = 14.30%

(c)

Variable Cost per unit (Rs.)	BEP	Increase in variable cost	Increase in BEP
35,00,000 40,00,000	1,25,000 units 1,35,659units	(24.20 – 22)*100 / 22 = 4.21%	(1,35,659 - 1,25,000)*100/ 1,25,000 = 9%

Observation:

- 20% increase in selling price results in 56% increase of BEP
- 14.29% increase in fixed cost results in 14.30% increase in BEP
- 10% increase in variable cost results in 9% increase of BEP

Out of three factors (selling price, fixed cost, variable cost), **BEP is more sensitive to selling price.**

8.6. Risk adjusted MARR

Uncertainty causes factors inherent to engineering economy studies, such as cash flows and project life, to become random variables in the analysis. A widely used industrial practice for including some consideration of uncertainty is to increase the MARR when a project is thought to relatively uncertain. Hence a procedure has emerged that employs risk adjust interest rates. Though this method for dealing with risk is very simple, it is not generally recommended as it has the following shortcomings:

(a) In the name addressing risk, the interest rate is adjusted to upper limit which makes or may make cost only project appear more attractive.

(b) Alternative having the lowest investment seem favorable at higher interest rates irrespective of its future cash flows.

(c) Since we are expecting higher rate of return from risky alternatives, we are penalizing the alternative in some way. But at this higher value of interest rate, the equivalent worth will also be higher for this risky alternative and based on the selection criterion, the same would be selected which we intended to avoid.

Example 6.8

The atlas corporation is considering two alternatives, both affected by uncertainty of different degrees, for increasing the recovery of a precious metal its smelting process. The following data concern capital investment requirements and estimated annual savings of both alternatives. The firm's MARR for its risk free investments is 10% per year.

	Alternative	
End of Year	P	Q
0	-1,60,000	-1,60,000

1	1,20,000	20,827
2	60,000	60,000
3	0	1,20,000
4	60,000	60,000

Risk adjusted MARR for Alternative P = 20%
Risk adjusted MARR for Alternative Q = 17%

With the above information regarding uncertainty of these alternatives, which one would be the most economical?

Solution

Using PW formulation

PW_P (20%) = -1, 60,000 + 1, 20,000 (P/F, 20%, 1) + 60,000 (P/F, 20%, 2) + 60,000 (P/F, 20%, 4)

= 10,602

PW_Q(17%) = -1,60,000 + 20,827 (P/F, 17%, 1) + 60,000 (P/F, 17%, 2) + 1,20,000 (P/F, 17%, 3) +60,000 (P/F, 17%, 4)

= 8,575

Here, PW_P (20%) > PW_Q (17%), **select Alternative P**

Optimistic- pessimistic – most likely estimation (Scenario Analysis)

The expected cash flows that we use to value risky assets can be estimated in one or two ways. They can represent a probability-weighted average of cash flows under all possible scenarios or they can be the cash flows under the most likely scenario. While the former is the more precise measure, it is seldom used simply because it requires far more information to compile. In both cases, there are other scenarios where the cash flows will be different from expectations; higher than expected in some and lower than expected in others. In scenario analysis, we estimate expected cash flows and asset value under various scenarios, with the intent of getting a better sense of the effect of risk on value.

8.7 Decision Tree

Alternative evaluation may require a series of decisions where the outcome from one stage is important to the next stage of decision making. When each alternative is clearly defined and probability estimates can be made to account for risk, it is helpful to perform the evaluation using decision tree. "Decision tree is a powerful means of facilitating the analysis of important problems, especially those that involve sequential decisions and variable outcomes over time.

A decision tree includes:

- More than one stage of alternative selection.
- Selection of an alternative at one stage that leads to another stage.
- Expected results from a decision at each stage.
- Probability estimates for each outcome.
- Estimate of economic value (cost or revenue) for each outcome.
- Measure of worth as the selection criterion such as E (PW).

Components of Decision Tree

- The decision tree is constructed left to right and includes each possible decision and outcome.
- **Decision node**: A square represents a decision node for making decision by a decision maker.
- **Branch**: It is a line connecting nodes from the left to the right of the diagram.
- **Probability node**: A circle represents probability node with the possible outcomes and estimated probabilities on the branches.

For the Evaluation and Selection of the alternative, the following information is necessary.

- The probability that is estimated must sum to 1.0 for each set of outcomes (branches) that results from decision.

- Economic information for each decision alternative and possible outcome, such as initial investment and estimated cash flows.

Procedure for Solving Decision Tree Using PW analysis

- Start at the top right of the tree. Determine the PW value for each outcome branch.
- Calculate the Expected value for each decision alternative

  E (decision) = Σ (outcome estimate) P (outcome)
- At each decision node, select the best E (decision) value – minimum cost or maximum value (if both cost and revenues are estimated).
- Continue moving to the left of the tree to the root decision in order to select the best alternative.

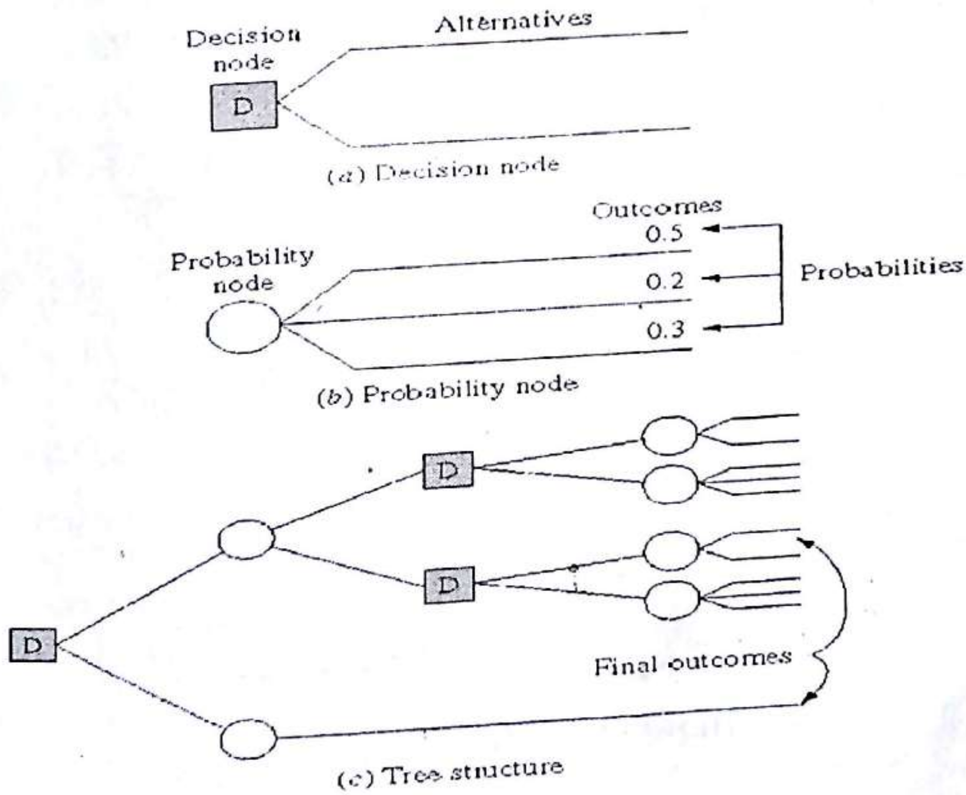


Fig: Decision tree diagram showing decision node, probability node and tree structure

Example 8.9

A company is mechanizing its packaging plants by investing Rs 3, 60,000. Estimates for efficiency of design goals, their probabilities and corresponding annual expenses saving are as follows:

Design goal met (%)	Probability	Annual expenses saving (Rs)
85	0.35	40,000
60	0.50	33,000
50	0.15	26,000

The Company has another option to invest this money that earns 20% per year elsewhere. Based on the Expected PW as the decision criterion, determine whether the mechanizing is preferable or not? MARR = 12% and analysis period is 5 years.

Solution

The PW corresponding to each efficiency goals are:

PW (12%) 85 = - 3, 60,000 + 40,000 (P/A, 12%, 5) = - Rs 2, 15,808

PW (12%) 60 = - 3, 60,000 + 33,000 (P/A, 12%, 5) = - Rs 2, 41,042

PW (12%) 50 = - 3, 60,000 + 26,000 (P/A, 12%, 5) = - Rs 2, 66,275

For Option 2 (Invest elsewhere that earns 20% per year)

Annual earning = (0.2 * 3, 60,000) = Rs 72,000

PW (12%) = - 3, 60,000 + 72,000 (P/A, 12%, 5) = - Rs 1, 00,454

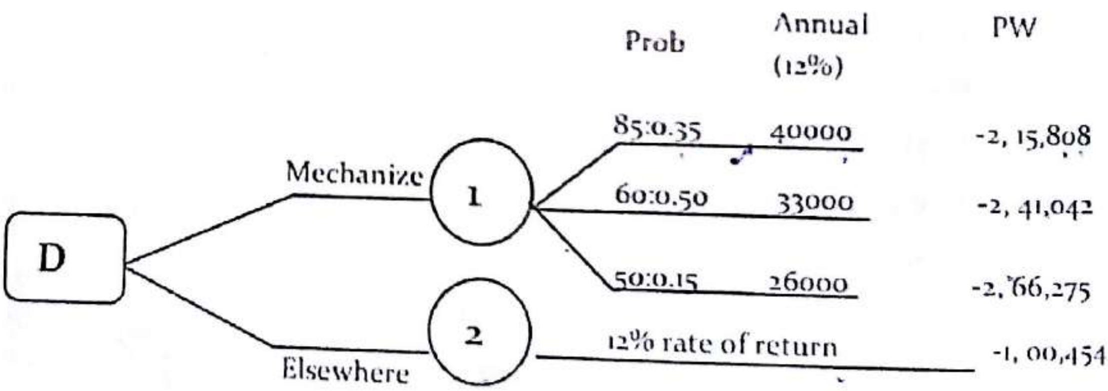


Fig: The single state decision tree diagram

Expected PW

Option1

E (PW) = - 2, 15,808 (0.35) – 2, 41,042 (0.50) – 2, 66,275 (0.15)

= - Rs 2, 35,995

Option 2

E (PW) = - Rs 1, 00,454

The analysis indicates that none of the options are preferable and even if we make the decision, the **option 2 should be selected.**

Example 8.10

A decision is needed to either market or sell a new invention. If the product is marketed, the next decision is to take it international or national. Assume the details of the outcome branches result in the decision tree of the given figure. The probability of each outcome and PW of CFBT are indicated. These payoffs are in millions of Rupees. Determine the best decision at the decision node D1.

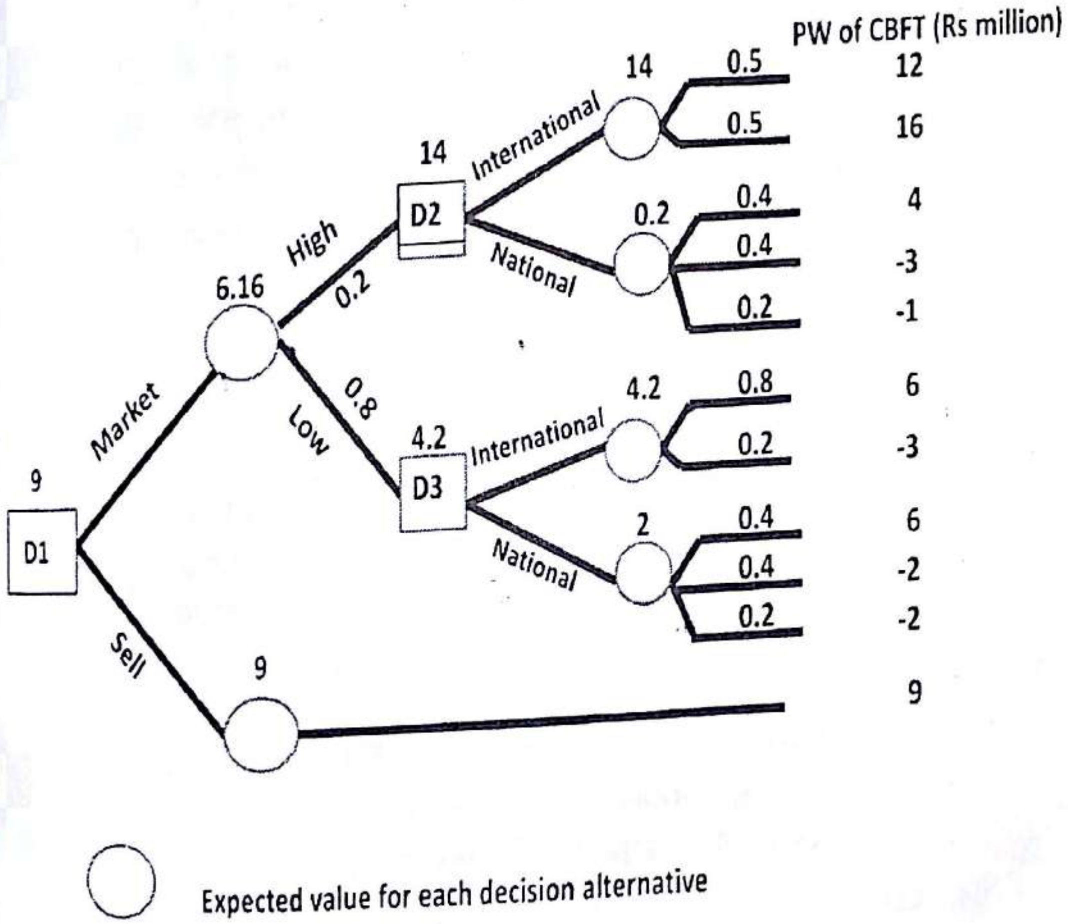


Fig: Two stage decision tree diagram

Solution

- Present worth of CFBT is supplied.

- Calculate the expected PW values for alternatives from node D2

Expected value (international) = 12 *0.5 + 16 * 0.5 = 14
Expected value (national) = 4*(0.4)-3*(0.4)-1*(0.2)
= 0.2

- The expected PW values of 4.2 and 2 for D3 are calculated in the similar fashion.
- Select the larger expected value at each decision node. These are 14 (international) at D2 and 4.2 (international) at D3.
- Calculate the expected PW for the two D1 branches

Expected value (market decision) = 14 *0.2 +4.2* 0.8
= 6.16

Expected value (sell) = 9 *1.0 = 9

- The sell decision yields the larger expected PW value of 9.
- The larger expected PW of CBFT path is to select the sell branch at D1 for guaranteed Rs. 9,000,000.

Some solved Examples

1. Calculate breakeven volume of a cable manufacturing company from the following data. Total Cost = Rs. 1,200,000, variable cost = Rs. 400,000. Income from sales = 15, 00,000 at production of 5000 units. **(TU,IOE,2069)**

Solution

Total Cost (TC) = Fixed Cost (FC) + Variable Cost (VC)
FC = 1,200,000 – 400,000 = 800000
Variable cost per unit (v) = VC/Q = 400000/5000 = 80
Sales cost per unit (s) = total sales / Q
= 1500000/5000 = 300
Break Even Volume = FC/(s-v) = 800000/ (300-80)
= 3636.36 units

2. A proposal is described by the following estimates; P = $20000, S =0, N =5 and net annual receipts = $ 7000. A rate of return of 20 percent is desired on such

proposals. Construct a sensitivity graph of the life, annual receipts, and rate of return for deviations over a range of ±20 percent. To which element is the decision most sensitive?

Solution

Step I: Developing the prime equation using PW formulation

PW (20%) = - 20000 + 7000 (P/A, 20%, 5) + 0
= -20000 + 7000 * 2.9906
= 934.2

Step II: Varying Annual receipt (A) by ±20 percent

PW (20%) = - 20000 + 7000 (1±20%) (P/A, 20%,5)

Taking +10%

-20000 + 7000 (P/A, 20%, 5) + 7000 *0.1(P/A, 10%, 5)

934.2 + 7000*0.1 * 2.9906 = 3027.62

Taking +20%

-20000 + 7000 (P/A, 20%, 5) + 7000 *0.2(P/A, 10%, 5)

934.2 + 7000*0.2 * 2.9906 = 5121.04

Taking -10%

-20000 + 7000 (P/A, 20%,5) - 7000 *0.1(P/A, 10%, 5)

934.2 - 7000*0.1 * 2.9906 = -1159.22

Taking -20%

-20000 + 7000 (P/A, 20%, 5) - 7000 *0.2(P/A, 10%, 5)

934.2 - 7000*0.2 * 2.9906 = -3252.64

Step III: Varying useful life (N) by ±20 percent

Taking +10%, N = 5.5

- 20000 + 7000 (P/A, 20%, 5.5) + 0

$-20000 + 7000 \{ ((1+ 0.2)^{5.5} -1)/ ((1+ 0.2)^{5.5} * 0.2)\}$

-20000 + 7000 * 3.165 = 2159.81

Taking +20%, N = 6

- 20000 + 7000 (P/A, 20%, 6) + 0

$-20000 + 7000 \{(1+0.2)^6 -1)/(1+0.2)^6 *0.2)\}$

$-20000 + 7000 * 3.3255 = 3278.5$

Taking -10% , N = 4.5

$- 20000 + 7000 (P/A, 20\%, 4.5) + 0$

$-20000 + 7000 \{(1+0.2)^{4.5} -1)/(1+0.2)^{4.5} *0.2)\}$

$-20000 + 7000 * 2.7988 = -408.21$

Taking -20% , N = 4

$- 20000 + 7000 (P/A, 20\%, 4) + 0$

$-20000 + 7000 \{ (1+0.2)^4 -1)/(1+0.2)^4 *0.2)\}$

$-20000 + 7000 * 2.5887 = -1879.1$

Step IV: Varying rate of return (i) by ±20 percent

Taking + 10% , i=22%

$-20000 + 7000 (P/A, 22\%, 5)$

$-20000 + 7000 \{ (1+0.22)^5 -1)/(1+0.22)^5 *0.22)\}$

$-20000 + 7000 * 2.8636 = 45.2$

Taking + 20% , i=24%

$-20000 + 7000 (P/A, 24\%, 5)$

$-20000 + 7000 \{(1+0.24)^5 -1)/(1+0.24)^5 *0.24)\}$

$-20000 + 7000 * 2.745 = -782.30$

Taking - 10% , i=18%

$-20000 + 7000 (P/A, 18\%, 5)$

$-20000 + 7000 \{ (1+0.18)^5 -1)/(1+0.18)^5 *0.18)\}$

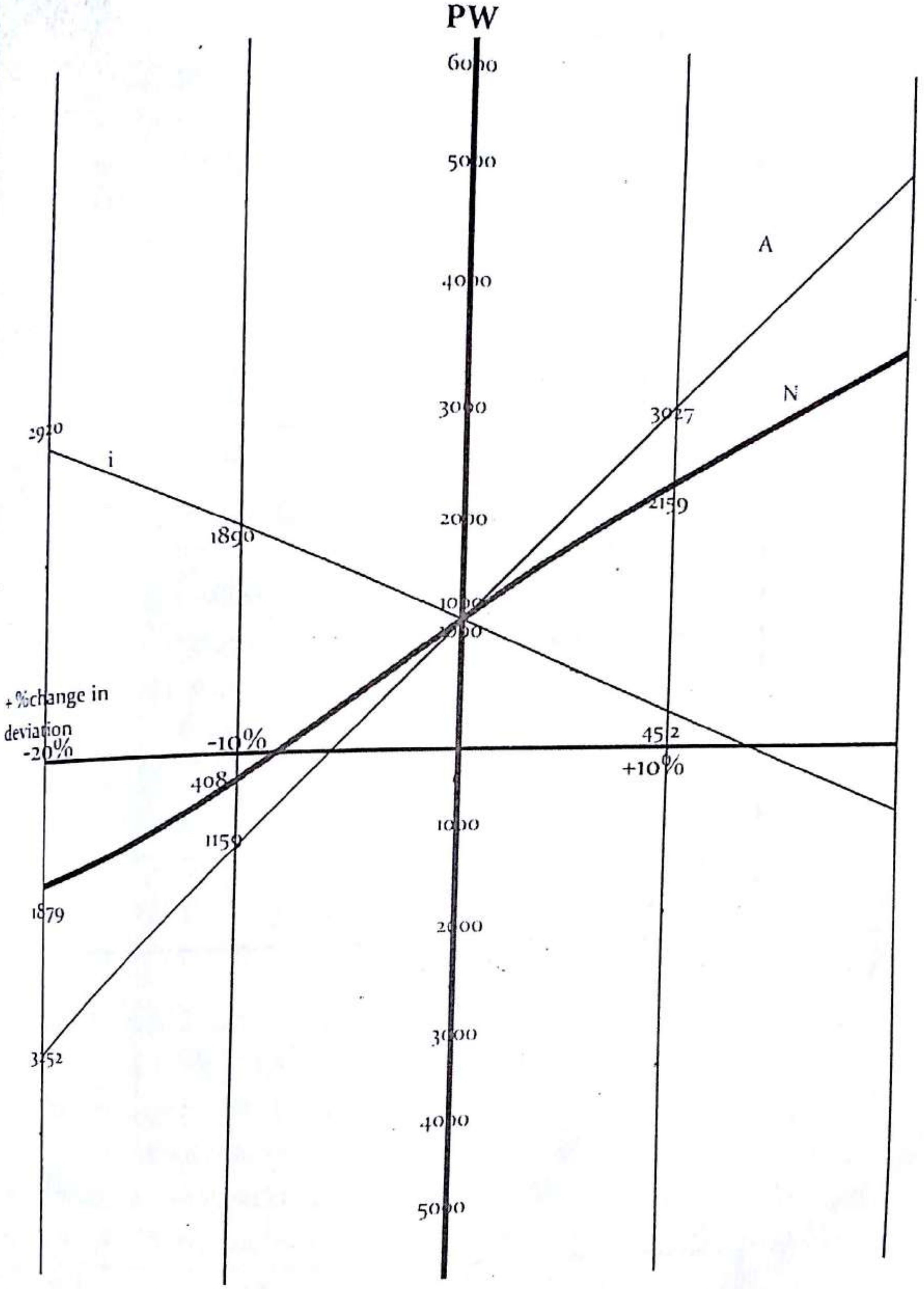
$-20000 + 7000 * 3.1272 = 1890.4$

Taking - 20% , i=16%

$-20000 + 7000 (P/A, 16\%, 5)$

$-20000 + 7000 \{(1+0.16)^5 -1)/(1+0.16)^5 *0.16)\}$

$-20000 + 7000 * 3.2743 = 2920.1$



From the graph it can be concluded that annual receipt (A) is more sensitive to PW

STUDENT LEARNING OBJECTIVE

From studying this chapter you will learn

- To understand the concept of inflation and terminologies associated with it.
- To measure the inflation
- To perform the economic equivalence calculation under inflation.
- Impact of inflation on Economic Evaluation

9.1 Meaning of Inflation

Item	Year 2055 B.S.	Year 2070 B.S.
Monthly household expenses	Rs. 10000	Rs. 25000
Cigarette per box	Rs. 50	Rs. 110
Petrol per liter	Rs. 70	Rs. 123
Men's T shirt	Rs. 450	Rs. 900

The table demonstrates price difference between 2055 and 2070 for some commonly bought items. Example, cigarette costs Rs 50 in 2055, whereas it cost Rs 110 in 2070. In 2070, the same Rs 50 bought only fraction of cigarette it would have bought in 2055 (about 45.45% of cigarette (50/110). From 2055 to 2070, the Rs. 50 sum had lost 58.83% of its purchasing power. The cost of an item tends to increase over time, OR, the same dollar amount buys less of an item over time, which is called inflation In other words, a loss in the purchasing power of money over time is inflation. When the net demand for services and goods exceeds the total supply, because of less supply, the net prices of these services/goods increases and this kind of situation is known as inflation. Inflation produces effects on the health of an economy. For example- the uncertainty about the future behavior of "inflation rate" may restrict the investors to invest in the market. If consumers of an economy came to know that prices are going to increase in *coming future* then they will over purchase the goods (in the *fascination* of present lower prices), this will cause shortage of *various goods* in the market. Because of inflation rupees in one *period* of time are not equivalent to rupees in another. We *know* that engineering economic analysis requires that *comparisons* be made on an equivalent basis. So it is important *for us* to be able to incorporate the effects of inflation.

Deflation is the opposite of inflation in that price usually *decreases* over time, and hence, a specified Rupees amount *gains* in purchasing power. Inflation is far more common than *deflation* in real world. Our focus will be restricted to *accounting* for inflation in economic analysis.

Reasons/Causes of Inflation

1. If the production cost of various services and goods increases then naturally the prices of the final products would also increase.
2. When industries and business houses increase the total prices of their services and goods in order to amplify their profit margins.
3. When a specific section of a mass industry increases the prices of its services and goods, because this step of a particular section of a mass industry will produce considerable effects on various other sections of industry also. For example- increase in the price of crude oil will spontaneously cause increase in the bus fares and airfares.
4. *Hyperinflation* occurs during or after a heavy war.
5. Another severe type of inflation is known as *stagflation*. It occurs in an economy which faces economic stagnation and high unemployment rate.

Terminologies used in Inflation

To introduce the effect of inflation into our economic analysis, we need to define several inflation related terms.

1. **Consumer Price Index (CPI)**

The CPI measures average change in the prices paid for food, shelter, medical care, transportation and other goods and services used by individual or families.

2. **Producer Price index (PPI)**

The PPI is good measure of the industrial price increases.

When performing the engineering economic analysis, the appropriate price indexes must be selected to estimate the price increase of raw materials, finished products and operating costs.

3. **Actual (current) / Future dollars (A$)**

Actual dollars are estimates of future cash flow for year 'n' that take into account any anticipated changes in amount due to inflationary or deflationary effects. Usually these amounts are determined by applying an inflation rate to base year dollar estimates.

This means inflation rate is constant

4. **Constant (real)/ Base year dollars (A'$)**

Constant dollar represents constant purchasing power independent of the passage of time. In situations where inflationary effects were assumed when cash flows were estimated, these estimates can be converted to constant dollar (base year dollars) by adjustment using some accepted general inflation rate.

5. **Average inflation rate (f)**

A measure of the average change in the purchasing power of a dollar during a specified period of time. For the effect varying yearly inflation rates over a period of several years, we can compute a single rate that represents average inflation rate.

Since each individual year's inflation rate is based on the previous year's rate, these rates have a compounding effect.

9.2 Measuring Inflation

Let's suppose we want to calculate the average inflation rate for a 2-year period: the first year's inflation rate is 4%, and the second year's rate is 8%, using a base price of RS.100.

- Step 1: To find the price at the end of the second year we use the process of compounding.

First year

Rs 100 (1+0.04) (1+0.08) = Rs112.32

Second Year

- Step 2: to find the average inflation rate (f), we establish the equivalence relation.

$$\text{Rs. } 100\,(1+f)^2 = \text{RS. } 112.32$$

$$\text{Or, Rs. } 100\,(F/P, f, 2) = \text{Rs. } 112.32$$

Solving for f yields,

$$f = 5.98\% = 6\% \text{ (approximately)}$$

If the inflation rate (f) averages 6% over the next 10 years,

= Rs.100 (1+0.06)10

= Rs.179.08 = Rs. 180 (approximately)

It can be concluded that the

% increase in the price of the product

= (Rs 180 - Rs 100) / Rs 100

= 80%

This is an 80% increase over the 10 years before at 6% inflation rate.

Money in one period of time t_1 can be brought to the same value as money in another period of time t_2, by using the relation:

Future dollars or then-current dollars

$$\text{Dollars in period } t_1 = \frac{\text{Dollars in period } t_2}{\text{Inflation rate between t1 and t2}}$$

Today's dollars or Constant-value dollars

Example 9.1

Consider the following net cash flow of a project in a constant dollar.

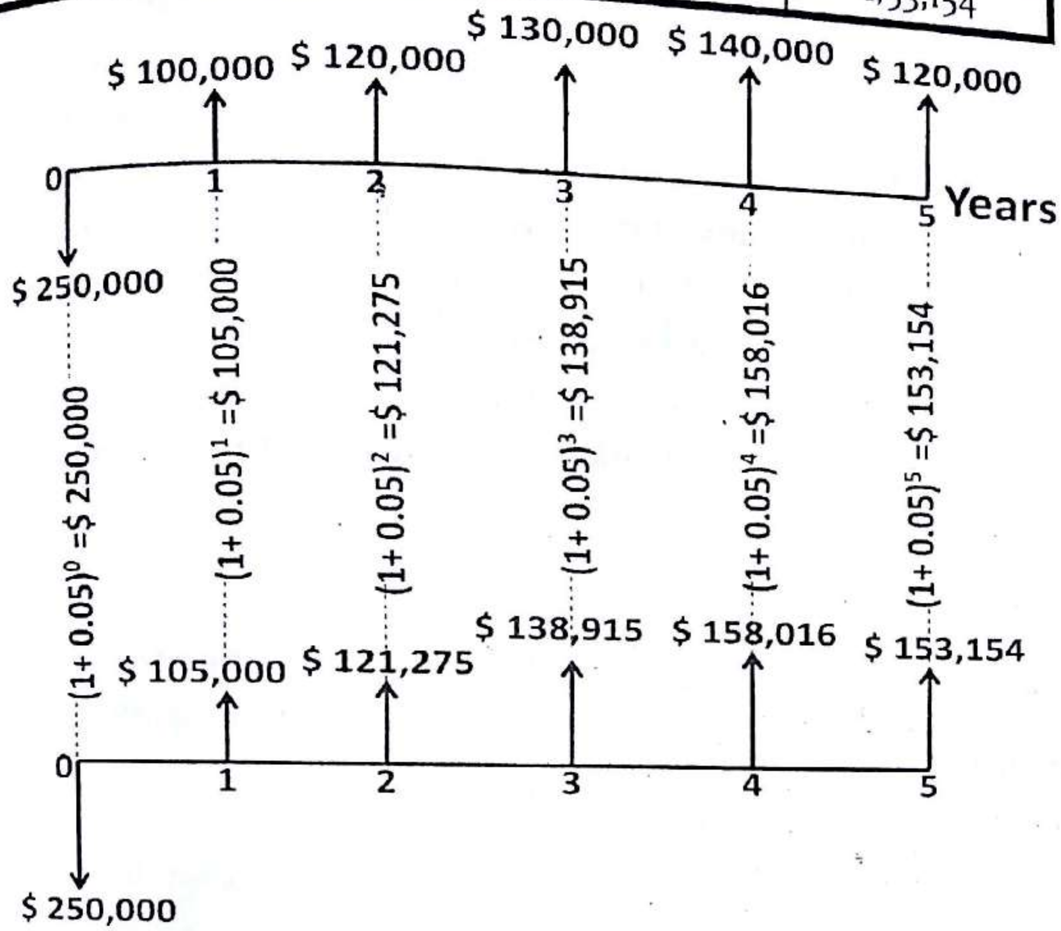
Period	Net Cash flow in Constant Dollar
0	-250,000
1	1,00,000
2	1,10,000
3	1,20,000
4	1,30,000
5	1,20,000

Assume that the price per unit as well as the manufacturing cost keeps up with the general inflation rate, which is projected to be 5% annually. Convert the project's cash flows into the equivalent actual dollars.

Solution

Period	Cash flow in Constant Dollar	Conversion Factor $= (1+f)^n$	Cash flow in Actual Dollar
0	-250,000	$(1+0.05)^0$	-250,000
1	1,00,000	$(1+0.05)^1$	1,05,000
2	1,10,000	$(1+0.05)^2$	1,21,275

3	1,20,000	$(1+0.05)^3$	138,915
4	1,30,000	$(1+0.05)^4$	1,58,016
5	1,20,000	$(1+0.05)^5$	1,53,154



9.3 Equivalence Calculation under Inflation

In previous chapters, our equivalence analysis took into consideration changes in earning power of money i.e. interest rate. To factor in changes in purchasing power i.e. inflation, we use either (1) constant dollar analysis or (2) actual dollar analysis. Either method produces the same result. Before the analysis of the procedures of these analyses, let's look at the different interest rate.

(a) Market /Inflation adjusted interest rate (i)

As its name implies, this is the interest rate that has been adjusted to take inflation into account. This rate takes into account the combined effect of the earning value of capital (earning power) and any anticipated inflation or deflation (purchasing power). Market interest rates are the rates stated by financial institutions (loans, bank accounts etc) and used by firms as their minimum acceptable rate of return (MARR) in evaluating their investments.

(b) **Inflation free interest rate (i')**

This rate is an estimate of the true earning power of the money when the inflation effect has been removed. This rate is commonly known as real interest rate. Although referred to as real interest rates, they are abstract (theoretical), but serve as a useful approach to what market rates might be if there were no inflation.

(c) **Inflation rate (f)**

This is the measure of the rate of change in the value of money. In calculating any cash flow equivalence, we need to identify the nature of the project cash flow.

Three common cases can be considered.

Case 1: All cash flow elements are estimated in constant dollars.

Case 2: All cash flow elements are estimated in actual dollars.

Case 1: If cash flow elements are estimated in Constant Dollar (Constant Dollar Analysis)

- Suppose that all cash flow elements are already given in constant dollar, and that we want to compute the equivalent PW of the constant dollar (A'n) occurring in year 'n'.
- In the absence of inflationary effect, we use inflation free interest rate (i') to account only for the earning power of money.

- The equivalent present worth is obtained by

$$Pn = A'n / (1 + i)^n$$

Example 9.2
Consider the constant dollar cash flows for a company as follows. If the manager wants the company to earn a 12% inflation free rate of return before tax on any investment, what should be the present worth of this project?

Period	Net Cash flow in Constant Dollar
0	-250,000
1	1,00,000
2	1,10,000
3	1,20,000
4	1,30,000
5	1,20,000

Solution
Since all values are in constant dollars, we can use the inflation free interest rate. We simply discount the dollar inflows at i =12%

PW (12%) = - 250,000 + 100,000 (P/A, 12%, 5) + 10,000 (P/G, 12%, 4) + 20,000 (P/F, 12%, 5)
PW (12%) = 163,099
Since the equivalent net receipts exceed the investment, the project can be justified before considering any tax effects.

Case 2: If cash flow elements are estimated in Actual Dollar (Actual Dollar Analysis)

- In this method, all cash flow elements are estimated in actual dollars.

- To find the equivalent present worth of this actual dollar amount (An) in year n, we may use either *Deflation method* or the *Adjusted-Discount method.*

Deflation Method

- It requires two steps to convert actual dollars into equivalent present worth dollars.
- First, we convert actual dollars into equivalent constant dollars by discounting by the general inflation rate which removes the inflationary effect.
- Second, finding the equivalent present worth using i%.

Example 9.3

An electronics manufacturing company is investing to produce control systems. The project is expected to generate the following cash flow in actual dollars.

Period	Net Cash flow in Actual Dollar
0	-75,000
1	32,000
2	35,700
3	32,800
4	29,000
5	58,000

(a) What are the equivalent year of dollars (constant dollars), if the general inflation rate (f) is 5% per year?

(b) Compute the present worth of these cash flows in constant dollars at i = 10%.

Solution

The net cash flows in actual dollars can be converted to constant dollars by deflating them by 5% yearly deflation factor. The deflated or constant dollar cash flows can then be used to determine NPW at i%.

(a) Convert the actual dollars into constant dollars as follows:

Period	Cash flow in Actual Dollar	Deflation Factor $= (1+f)^{-n}$	Cash flow in Constant Dollar
0	-75,000	$(1+0.05)^{-0}$	-75,000
1	32,000	$(1+0.05)^{-1}$	30,476
2	35,700	$(1+0.05)^{-2}$	32,381
3	32,800	$(1+0.05)^{-3}$	28,334
4	29,000	$(1+0.05)^{-4}$	23,858
5	58,000	$(1+0.05)^{-5}$	45,445

(b) Compute the equivalent present worth of constant dollar using i =10%

Period	Cash flow in Actual Dollar	Discounting Factor $= (1+f)^{-n}$	Cash flow in Constant Dollar
0	-75,000	$(1+0.1)^{-0}$	-75,000
1	30,476	$(1+0.1)^{-1}$	27,706
2	32,381	$(1+0.1)^{-2}$	26,761
3	28,334	$(1+0.1)^{-3}$	21,288
4	23,858	$(1+0.1)^{-4}$	16,295
5	45,445	$(1+0.1)^{-5}$	28,218
Total Present Worth			45,268

Adjusted – discount Method

The two-step process shown in previous example can be streamlined by the efficiency of the adjusted-discount method, which performs deflation and discounting in one step.

Mathematically,

We can combine this two step procedure into one step

$$P_n = \frac{\frac{A_n}{(1+f)^n}}{(1+i')^n}$$

$$P_n = \frac{A_n}{(1+f)^n (1+i')^n}$$

$$P_n = \frac{A_n}{\{(1+f)(1+i')\}^n} \quad \text{..........} \quad (1)$$

Since the market interest rate (i) reflects both the earning power and the purchasing power, we have following relation relationship

$$P_n = \frac{A_n}{(1+i)^n} \quad \text{..................} \quad (2)$$

The equivalent present worth values in equation (1) and (2) must be equal at year 0.

Therefore,

$$\frac{A_n}{\{(1+f)(1+i')\}^n} = \frac{A_n}{(1+i)^n}$$

This leads to the following relationship among f, i', i

$$(1+i) = (1+f)(1+i')$$

Simplifying the term yields,

$$i = i' + i' * f + f$$

Example 9.3

Consider the cash flows in actual dollar as in previous example as follows. Compute the equivalent present worth of these flows using the adjusted discount method. f = 5%, and i = 10%

Period	Net Cash flow in Actual Dollar
0	-75,000
1	32,000
2	35,700
3	32,800
4	29,000
5	58,000

Solution

First, determine the market interest rate (i)

$$i = i' + i' * f + f$$

$$i = 0.10 + 0.10 * 0.05 + 0.05 \text{ i}$$

$$i = \text{i} = 15.5\ \%$$

Period	Cash flow in Actual Dollar	Multiplied by = $(1+0.155)^{-n}$	Cash flow in Constant Dollar
0	-75,000	$(1+0.155)^{-0}$	-75,000
1	32,000	$(1+0.155)^{-1}$	27,706
2	35,700	$(1+0.155)^{-2}$	26,761
3	32,800	$(1+0.155)^{-3}$	21,288
4	29,000	$(1+0.155)^{-4}$	16,295
5	58,000	$(1+0.155)^{-5}$	28,218
Total Present Worth			**45,268**

The equivalent present worth that we obtained using adjusted - discount method (i=15.5%) is exactly same as the result obtained using the deflation method (f=5% and i=10%)

9.4 Impact of Inflation on Economic Evaluation

When Inflation is modest, 2 to 4 percent per year, it is generally ignored in economic evaluations of proposals. It is argued that all proposals are affected similarly by price changes and that there are too little differences between current and future costs to influence the order of preference. This argument is not applicable when inflation is high and some goods and services escalate much more rapidly than others. Once analysts recognize that inflation has an impact on most investment opportunities and therefore deserves consideration in their appraisals, they must decide on the most appropriate method in which to include it. There are two basic methods considered:

1. Estimate inflation effects by converting all cash flows to money units that have constant purchasing power, called constant (real) dollars. This approach is most

suitable for before tax analysis, when all cash flow components inflate at uniform rates.

2. Estimate cash flows in the amount of money units actually exchanged at the time of each transaction. These money units are called actual (future) dollars. The actual dollar approach is easier to understand and apply and is more versatile than the constant dollar method.

In an analysis by either method, it is critical that the interest rate used in the analysis corresponds to the assumptions made in determining the cash flows. Two different interest rates are used in economic calculations: market interest rate (i) and inflation free interest rate (i').

Example 9.4

Suppose that you want to invest some amount of money for your individual purpose. Currently, the bank is paying a rate of 5.5% compounded annually. Assume inflation is expected to be 2% per year. Identify i, f, and i'.

Solution

If inflation is 2% per year,

We know ,

$$i = i' + i' * f + f$$

$$i - f = i' (1 + f)$$

$$i' = (i - f)/ (1 + f)$$

$$i' = \frac{(0.055 - 0.02)}{(1 + 0.02)}$$

$$i' = 0.034 = 3.4\%$$

Example 9.5

A XYZ company is interested in evaluating a major new mobile technology. Two competing companies (A and B) have approached the XYZ to develop the technology. XYZ Company believes that both companies will be able to deliver equivalent products at the end of 5 year period. Determine which company XYZ should choose if the MARR is 25% and price inflation is assumed to be 3.5% per year over the next 5 years.

Company A costs: development costs will be $ 150,000 the first year and will increase at a rate of 5% over the 5-year period. Company B costs: development costs will be a constant $ 150,000 per year in terms of today's dollar over the 5-year period.

Solution

The cost for each of the two alternatives is as follows:

Year	Future $ stated by A	Constant $ stated by B
1	$150{,}000\ (1+0.5)^{0} = 150{,}000$	150,000
2	$150{,}000\ (1+0.5)^{1} = 157{,}500$	150,000
3	$150{,}000\ (1+0.5)^{2} = 165{,}375$	150,000
4	$150{,}000\ (1+0.5)^{3} = 173{,}644$	150,000
5	$150{,}000\ (1+0.5)^{4} = 182{,}326$	150,000

We inflate the stated year cost given by Company A by 5% per year to obtain the actual dollars each year. Company B's costs are given in terms of constant dollars.

Using Constant Dollar Analysis

Converting the actual dollar of company A to the Constant dollar by using the deflation factor

Year	Future $ stated by A	Constant $ stated by B
1	$150{,}000\ (1+0.5)^{-1} = 144{,}928$	150,000
2	$150{,}000\ (1+0.5)^{-2} = 147{,}028$	150,000
3	$150{,}000\ (1+0.5)^{-3} = 149{,}159$	150,000
4	$150{,}000\ (1+0.5)^{-4} = 151{,}321$	150,000
5	$150{,}000\ (1+0.5)^{-5} = 153{,}514$	150,000

We use the real interest rate (i') to calculate the present worth of costs for each alternative.

$$i = i' + i' * f + f$$

$$i - f = i'\ (1 + f)$$

$$i' = (i - f) / (1 + f)$$

$$i' = \frac{(0.25 - 0.035)}{(1 + 0.035)}$$

$$i' = 0.208 = 20.8\%$$

PW of costs (A) = 144,928 (P/F, 20.8%, 1) + 147,028 (P/F, 20.8%, 2) + 149,159 (P/F, 20.8%, 3) + 151,321 (P/F, 20.8%, 4) + 153,514 (P/F, 20.8%, 5) = **$ 436,000**

PW of costs (B) = 150,000 (P/A, 20.8%, 5)
= 150,000 (2.9387) = **$ 441,000**

<u>Using Actual Dollar Analysis</u>

Year	Future $ stated by A	Constant $ stated by B
1	$150{,}000\ (1+0.5)^0 = 150{,}000$	$150{,}000\ (1+0.035)^1 = 155{,}250$
2	$150{,}000\ (1+0.5)^1 = 157{,}500$	$150{,}000(1+0.035)^2 = 160{,}684$
3	$150{,}000\ (1+0.5)^2 = 165{,}375$	$150{,}000(1+0.035)^3 = 166{,}308$
4	$150{,}000\ (1+0.5)^3 = 173{,}644$	$150{,}000(1+0.035)^4 = 172{,}128$
5	$150{,}000\ (1+0.5)^4 = 182{,}326$	$150{,}000(1+0.035)^5 = 178{,}153$

PW of costs (A) = 150,000 (P/F, 25%, 1) + 157,500 (P/F, 25%, 2) + 165,375 (P/F, 25%, 3) + 173,644 (P/F, 25%, 4) + 182,326 (P/F, 25%, 5) = **$ 436,000**

PW of costs (B) = 155,250 (P/F, 25%, 1) + 160,684 (P/F, 25%, 2) + 166,308 (P/F, 25%, 3) + 172,128 (P/F, 25%, 4) + 178,153 (P/F, 25%, 5) = **$ 441,000**

Using either constant dollar or actual dollar, XYZ should Choose Company A offer.

Review Question

1. Define inflation in terms of purchasing power.
2. How does inflation happen? Describe a few circumstances that causes price in an economy to increase.
3. Explain Constant dollar and actual dollar analysis?
4. What do you understand by market interest rate, inflation free interest rate, and inflation rate?